Straight Line

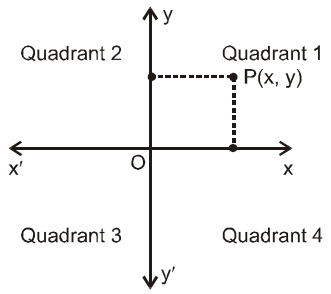
The knowledge of which geometry aims is the knowledge of the eternal..... Plato

A french mathematician and a greatest philosopher named Rene Descartes, pioneered the use of algebra in Geometry. He suggested methods to study geometry by algebraic methods without making direct reference to the actual figures

This geometry was called co-ordinate geometry or analytical geometry and it is the branch of geometry in which algebraic equations are used to denote points, lines and curves.

Rectangular cartesian co-ordinate systems :

We shall right now focus on two-dimensional co-ordinate geometry in which two perpendicular lines called co-ordinate axes (x-axis and y-axis) are used to locate a point in the plane.



O is called origin. Any point P in this plane can be represented by a unique ordered pair (x, y), which are called co-ordinates of that point. x is called x co-ordinate or abscissa and y is called y co-ordinate or ordinate. The two perpendicular lines xox′ and yoy′ divide the plane in four regions which are called quadrants, numbered as shown in the figure.

Let us look at some of the formulae linked with points now.

Distance Formula :

In rectangular Cartesian coordinate system

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is = $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

Example # 1 : Find the value of x, if the distance between the points (x, 8) and (4, 3) is 13

Solution : Let P(x ,8) and Q(4, 3) be the given points. Then PQ = 13 (given)

$\sqrt{(x-4)^2+(8-3)^2} = 13 \quad \Rightarrow \quad (x-4)^2 + 25 = 169 \Rightarrow x = 16$ or $x = -8$

Self practice problems :

(1) Show that four points (0, –1), (6, 7) (–2, 3) and (8, 3) are the vertices of a rectangle.

(2) Find the co-ordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, –2) and (2, –2). Also find its circumradius.

Ans. (2) (5, 2), 5

Section Formula :

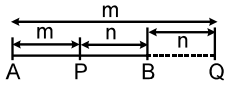
If P(x, y) divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then;

$x = \frac{mx_2 + nx_1}{m + n}$; $y = \frac{my_2 + ny_1}{m + n}$.

Notes : (i) If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

(ii) If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically, $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

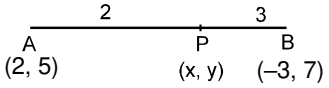


Example # 2 : Find the co-ordinates of the point which divides the line segment joining the points (2, 5) and (– 3, 7) in the ratio 2 : 3 (i) internally and (ii) externally.

Solution : Let P (x, y) be the required point.

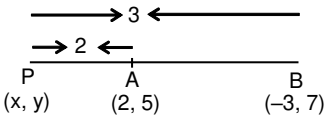
(i) For internal division :

$$(x, y) = \left(\frac{-6+6}{2+3}, \frac{14+15}{2+3}\right) = \left(0, \frac{29}{5}\right)$$



(ii) For external division

$$(x, y) = \left(\frac{-6-6}{2-3}, \frac{14-15}{2-3}\right) = (12, 1)$$



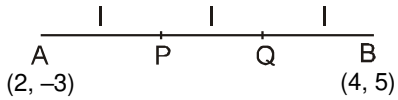
Example # 3 : Find the co-ordinates of points which trisect the line segment joining (2, – 3) and (4, 5).

Solution : Let A (2, –3) and B(4, 5) be the given points. Let the points of trisection be P and Q. Then AP = PQ = QB = λ (say)

$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = λ : 2λ = 1 : 2 and AQ : QB = 2λ : λ = 2 : 1

So P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1



So P divides AB internally in the ratio 1:2 while Q divides internally in the ratio 2: 1

$\therefore$ the co-ordinates of P are $\left(\frac{4+4}{1+2}, \frac{5-6}{1+2}\right) = \left(\frac{8}{3}, -\frac{1}{3}\right)$

and the co-ordinates of Q are $\left(\frac{8+2}{1+2}, \frac{10-3}{1+2}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$

Hence, the points of trisection are $\left(\frac{8}{3}, -\frac{1}{3}\right)$ and $\left(\frac{10}{3}, \frac{7}{3}\right)$

Self practice problems :

(3) In what ratio does the point (4, 1) divide the line segment joining the points (1, –2) and (5, 2).

(4) The three vertices of a parallelogram taken in order are (–2, 0), (4, 2) and (5, 3) respectively. Find the co-ordinates of the fourth vertex.

Ans. (3) 3 : 1 internally (4) (–1, 1)

The ratio in which a given line divides the line segment joining two points :

Let the given line ax + by + c = 0 divide the line segment joining A(x_1, y_1) & B(x_2, y_2) in the ratio m : n, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A & B are on the same side of the given line then m/n is negative but if A & B are on opposite sides of the given line, then m/n is positive

Example # 4 : Find the ratio in which the line joining the points A (1, 2) and B(– 3, 4) is divided by the line x + y – 5 = 0.

Solution : Let the line x + y = 5 divides AB in the ratio k : 1 at P

$\therefore$ co-ordinate of P are $\left(\frac{-3k+1}{k+1}, \frac{4k+2}{k+1}\right)$

Since P lies on x + y – 5 = 0

$\therefore$ $\frac{-3k+1}{k+1} + \frac{4k+2}{k+1} - 5 = 0 \quad \Rightarrow \quad k = -\frac{1}{2}$

$\therefore$ Required ratio is 1 : 2 externally. .

Aliter : Let the ratio is m : n

$\therefore$ $\frac{m}{n} = -\frac{(1\times1+1\times2-5)}{1\times(-3)+1\times4-5} = -\frac{1}{2}$ $\therefore$ ratio is 1 : 2 externally.

Self practice problem :

(5) If the line 2x– 3y + λ = 0 divides the line joining the points A (– 1, 2) & B(– 3, – 3) internally in the ratio 2 : 3, find λ.

Ans. $\frac{18}{5}$

Slope Formula :

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis. If θ = 0, then m = 0 & the line is parallel to the x-axis. If A (x_1, y_1) & B (x_2, y_2), $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by : $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.

Example # 5 : What is the slope of a line whose inclination with the positive direction of x-axis is :
(i) 30º (ii) 90º (iii) 135º

Solution : (i) Here θ = 30º

Slope = $\tan\theta = \tan 30º = \frac{1}{\sqrt{3}}$.

(ii) Here θ = 90º

$\therefore$ The slope of line is not defined.

(iii) Here θ = 135º

$\therefore$ Slope = $\tan\theta = \tan 135º = \tan(180º - 45º) = -\tan 45º = -1$.

Example # 6 : Find the slope of the line passing through the points :
(i) (2, 7) and (– 3, 4) (ii) (6, 9) and (–2, 7)

Solution : (i) Let A = (2, 7) and B = (– 3, 4)

$\therefore$ Slope of AB = $\frac{4-7}{-3-2} = \frac{3}{5}$ $\left(\text{Using slope} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

(ii) Let A = (6, 9), B = (–2, 7) $\therefore$ Slope of AB = $\frac{7-9}{-2-6} = \frac{1}{4}$

Self practice problems :

(6) Find the value of x, if the slope of the line joining (1, 5) and (x, –7) is 4.

(7) What is the inclination of a line whose slope is
(i) 0 (ii) 1 (iii) –1 (iv) $-1/\sqrt{3}$

Ans. (6) – 2 (7) (i) 0º, (ii) 45º, (iii) 135º, (iv) 150º

Condition of collinearity of three points :

Points A (x_1, y_1), B (x_2, y_2), C(x_3, y_3) are collinear if

(i) $m_{AB} = m_{BC} = m_{CA}$ i.e. $\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \left(\frac{y_2 - y_3}{x_2 - x_3}\right)$

(ii) $\Delta ABC = 0$ i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) AC = AB + BC or AB ~ BC

(iv) A divides the line segment BC in some ratio.

Example # 7 : Show that the points (–2, –1), (2, 7) and (5, 13) are collinear.

Solution : Let (–2, –1) (2, 7) and (5, 13) be the co-ordinates of the points A, B and C respectively.

Slope of AB = $\frac{7+1}{2+2} = 2$ and Slope of BC = $\frac{13-7}{5-2} = 2$

$\therefore$ Slope of AB = slope of BC

$\therefore$ AB & BC are parallel

$\therefore$ A, B, C are collinear because B is on both lines AB and BC.

Self practice problem :

(8) Prove that the points (a, 0), (0, b) and (1, 1) are collinear if $\frac{1}{a} + \frac{1}{b} = 1$

Area of a Triangle :

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, then its area is equal to

$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a $(-)$ ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

Note: Area of n-sided polygon formed by points (x_1, y_1) ; (x_2, y_2); .....;(x_n, y_n) is given by

$$\frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots\ldots\ldots\ldots \begin{vmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{vmatrix} + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right).$$

Here vertices are taken in order.

Example # 8 : If the co-ordinates of two points A and B are (2, 1) and (4, –3)respectively. Find the co-ordinates of any point P if PA = PB and Area of $\Delta PAB = 6$.

Solution : Let the co-ordinates of P be (x, y). Then

PA = PB $\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(x-2)^2 + (y-1)^2 = (x-4)^2 + (y+3)^2$ $\Rightarrow$ $x - 2y = 5$ ....(i)

Now, Area of $\Delta PAB = 6 \Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 4 & -3 & 1 \end{vmatrix} = \pm 6$ $\Rightarrow$ $4x + 2y - 10 = \pm 12$

$\Rightarrow$ $4x + 2y = 22$ or $4x + 2y = -2$ $\Rightarrow$ $2x + y = 11$ or $2x + y = -1$

Solving $2x + y = 11$ and $x - 2y = 5$ we get $x = \frac{27}{5}$, $y = \frac{1}{5}$.

Solving $2x + y = -1$ and $x - 2y = 5$, we get $x = \frac{3}{5}$, $y = -\frac{11}{5}$.

Thus, the co-ordinates of P are $\left(\frac{27}{5}, \frac{1}{5}\right)$ or $\left(\frac{3}{5}, -\frac{11}{5}\right)$

Self practice problems :

(9) The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

(10) The coordinates of A, B, C are (6, 3), (–3, 5) & (4,–2) respectively and p is any point (x, y).Show that the ratio of the areas of the triangles PBC and ABC is $\left|\frac{x+y-2}{7}\right|$

Ans. (9) $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Equation of a Straight Line in various forms :

Now let us understand, how a line can be represented with the help of an algebraic equation. A moving point (point with variable co-ordinates) is assumed on the line and a link is established between its co-ordinates with the help of some given parameters. There are various forms of lines depending on the specified parameter

Point - Slope form :

$y - y_1 = m (x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

Example # 9 : Find the equation of a line passing through (3, –4) and inclined at an angle of 150º with the positive direction of x-axis.

Solution : Here, m = slope of the line = $\tan 150º = \tan (90º + 60º) = -\cot 60º = -\frac{1}{\sqrt{3}}$, $x_1 = 3$, $y_1 = -4$

So, the equation of the line is $y - y_1 = m (x - x_1)$

i.e. $y + 4 = -\frac{1}{\sqrt{3}} (x - 3)$

$x + \sqrt{3}\, y + 4\sqrt{3} - 3 = 0$

Self practice problem :

(11) Find the equation of s a line passing through P(–3, 5) and whose slope is –2.
Ans. $2x + y + 1 = 0$

Slope-intercept form :

$y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis.

Example # 10 : Find the equation of a line with slope –3 and cutting off an intercept of 5 units on negative direction of y-axis.
Solution : Here $m = -3$ and $c = -5$. So, the equation of the line is $y = mx + c$
i.e. $y = -3x - 5$ or $3x + y + 5 = 0$

Self practice problem :

(12) Find the equation of a straight line which cuts off an intercept of length 3 on y- axis and whose slope is – 3.
Ans. $3x + y - 3 = 0$

Two point form : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

Example # 11 : Find the equation of the line joining the points (3, 4) and (–2, 5)
Solution : Here the two points are $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 5)$.
So, the equation of the line in two-point form is

$y - 4 = \frac{5-4}{-2-3} (x - 3) \quad \Rightarrow \quad x + 5y = 23$

Self practice problem :

(13) Find the equations of the sides of the triangle whose vertices are (–1, 8), (4, –2) and (–5, –3). Also find the equation of the median through (–1, 8)

Ans. $2x + y - 6 = 0$, $x - 9y - 22 = 0$, $11x - 4y + 43 = 0$, $21x + y + 13 = 0$

Determinant form : Equation of line passing through (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Example # 12 : Find the equation of line passing through (2, 4) & (– 1, 3).

Solution : $\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad x - 3y + 10 = 0$

Self practice problem :

(14) Find the equation of the passing through (– 2, 3) & (– 1, – 1).

Ans. $4x + y + 5 = 0$

Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

Example # 13 : Find the equation of the line which passes through the point (–3, 8) and the sum of its intercepts on the axes is 7.

Solution : Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

This passes through (–3, 8), therefore $-\frac{3}{a} + \frac{8}{b} = 1$ ....(ii)

It is given that $a + b = 7 \Rightarrow \quad b = 7 - a$.

Putting $b = 7 - a$ in (ii), we get $-\frac{3}{a} + \frac{8}{7-a} = 1$

$\Rightarrow \quad a^2 + 4a - 21 = 0 \Rightarrow \quad a = 3, -7$

For $a = 3$, $b = 4$ and for $a = -7$, $b = 14$

Putting the values of a and b in (i), we get the equations of the lines

$\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{-7} + \frac{y}{14} = 1$ or $4x + 3y = 12$ and $2x - y + 14 = 0$

Self practice problem :

(15) Find the equation of the line through (2, 3) so that the segment of the line intercepted between the axes is bisected at this point.

Ans. $3x + 2y = 12$.

Perpendicular/Normal form :

$x\cos\alpha + y\sin\alpha = p$ (where $p > 0$, $0 \le \alpha < 2\pi$) is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes an angle α with positive x–axis.

Example # 14 : Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the origin to the line makes an angle of 30º with the positive direction of the x-axis.

Solution : Here $p = 3$, $\alpha = 30^\circ$

$\therefore$ Equation of the line in the normal form is

$x\cos 30^\circ + y\sin 30^\circ = 3$ or $x\frac{\sqrt{3}}{2} + \frac{y}{2} = 3$ or $\sqrt{3}x + y = 6$

Self practice problem :

(16) The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction (clock-wise) of y-axis. Find the equation of the line.

Ans. $\sqrt{3}\,x - y + 14 = 0$

General Form : $ax + by + c = 0$ is the equation of a straight line in the general form

In this case, slope of line $= -\dfrac{a}{b}$

x - intercept $= -\dfrac{c}{a}$, y - intercept $= -\dfrac{c}{b}$

Example # 15 : Find slope, x-intercept & y-intercept of the line $2x - 3y + 5 = 0$.

Solution : Here, $a = 2, b = -3, c = 5$

$\therefore$ slope $= -\dfrac{a}{b} = \dfrac{2}{3}$

x-intercept $= -\dfrac{c}{a} = -\dfrac{5}{2}$

y-intercept $= \dfrac{5}{3}$

Self practice problem :

(17) Find the slope, x-intercept & y-intercept of the line $3x - 5y - 8 = 0$.

Ans. (17) $\dfrac{3}{5}, \dfrac{8}{3}, -\dfrac{8}{5}$

Parametric form :

$P(r) = (x, y) = (x_1 + r\cos\theta, y_1 + r\sin\theta)$ or $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ is the equation of the line in parametric form, where 'r' is the parameter whose absolute value is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line.

Remark : The above form is derived from point-slope form of line.

$y - y_1 = m(x - x_1)$ where $m = \tan\theta$ $\Rightarrow$ $y - y_1 = \dfrac{\sin\theta}{\cos\theta}(x - x_1)$

Example # 16 : Find the equation of the line through the point A(1, 4) and making an angle of 45º with the positive direction of x-axis. Also determine the length of intercept on it between A and the line $x + y - 10 = 0$

Solution : The equation of a line through A and making an angle of 45º with the x-axis is

$\dfrac{x-1}{\cos 45º} = \dfrac{y-4}{\sin 45º}$ or $\dfrac{x-1}{\frac{1}{\sqrt{2}}} = \dfrac{y-4}{\frac{1}{\sqrt{2}}}$ or $x - y + 3 = 0$

Suppose this line meets the line $x + y - 10 = 0$ at P such that AP = r. Then the co-ordinates of P are given by

$\dfrac{x-1}{\cos 45º} = \dfrac{y-4}{\sin 45º} = r$ $\Rightarrow$ $x = 1 + r\cos 45º, y = 4 + r\sin 45º$

$\Rightarrow$ $x = 1 + \dfrac{r}{\sqrt{2}}, y = 4 + \dfrac{r}{\sqrt{2}}$

Thus, the co-ordinates of P are $\left(1 + \dfrac{r}{\sqrt{2}}, 4 + \dfrac{r}{\sqrt{2}}\right)$

Since P lies on $x + y - 10 = 0$, so $1 + \frac{r}{\sqrt{2}} + 4 + \frac{r}{\sqrt{2}} = 10$

$\Rightarrow \quad 5 + \sqrt{2}\, r = 10 \Rightarrow r = \frac{5}{\sqrt{2}} \Rightarrow$ length $AP = |r| = \frac{5}{\sqrt{2}}$

Thus, the length of the intercept $= \frac{5}{\sqrt{2}}$.

Self practice problem :

(18) A straight line is drawn through the point $A\left(\sqrt{3}, 2\right)$ making an angle of $\pi/6$ with positive direction of the x-axis. If it meets the straight line $\sqrt{3x} - 4y + 8 = 0$ in B, find the distance between A and B.

Ans. 6 units

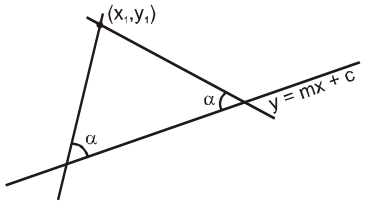
Angle between two straight lines in terms of their slopes:

If m_1 & m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \neq -1$) & θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Notes : (i) Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$, then the tangent of interior angles of the Δ ABC formed by these lines are given by, $\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}$; $\tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$ & $\tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$

(ii) The equation of lines passing through point (x_1, y_1) and making angle α with the line $y = mx + c$ are given by :

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.



Example # 17 : The acute angle between two lines is $\pi/4$ and slope of one of them is $-1/3$. Find the slope of the other line.

Solution : If θ be the acute angle between the lines with slopes m_1 and m_2, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Let $\theta = \frac{\pi}{4}$ and $m_1 \ -1/3$

$\therefore \quad \tan\frac{\pi}{4} = \left|\frac{-\frac{1}{3} - m_2}{1 - \frac{1}{3}m_2}\right| \quad \Rightarrow \quad 1 = \left|\frac{3m_2 + 1}{3 - m_2}\right| \quad \Rightarrow \quad \frac{3m_2 + 1}{3 - m_2} = 1 \text{ or } -1$

Now $\frac{3m_2 + 1}{3 - m_2} = 1 \quad \Rightarrow \quad m_2 = \frac{1}{2}$ and $\frac{3m_2 + 1}{3 - m_2} = -1 \quad \Rightarrow m_2 = -2.$

$\therefore$ The slope of the other line is either 1/2 or –2

Example # 18 : Find the equation of the straight line which passes through the point (3,–2) and making angle 60º with the line $\sqrt{3}\,x + y = 1$.

Solution : Given line is $\sqrt{3}\,x + y = 1$..

$\Rightarrow$ Slope of (1) $= -\sqrt{3}$.

Let slope of the required line be m. Also between these lines is given to be 60º.

$$\Rightarrow \quad \tan 60^\circ = \left|\frac{m-\left(-\sqrt{3}\right)}{1+m\,\left(-\sqrt{3}\right)}\right| \quad \Rightarrow \quad \sqrt{3} = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| \quad \Rightarrow \quad \frac{m+\sqrt{3}}{1-\sqrt{3}m} = \pm\sqrt{3}$$

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3} \quad \Rightarrow \quad m + \sqrt{3} = \sqrt{3} - 3m \quad \Rightarrow \quad m = 0$$

the equation of the required line is y + 2 = 0

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = -\sqrt{3} \quad \Rightarrow \quad m = \sqrt{3}$$

$\therefore$ The equation of the required line is $y + 2 = \sqrt{3}\,(x-3) \quad \Rightarrow \quad y\sqrt{3} = x - 2 - 3\sqrt{3}$

Self practice problem :

(19) A vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is x + y = 2. Find the equation of the other sides of the triangle.

Ans. (19) $(2-\sqrt{3})x - y + 2\sqrt{3} - 1 = 0$ and $(2+\sqrt{3})\,x - y - 2\sqrt{3} - 1 = 0$.

Parallel Lines :

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to y = mx + c is of the type y = mx + d, where 'd' is a parameter.

(ii) Two lines ax + by + c = 0 and a′x + b′y + c′ = 0 are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$.

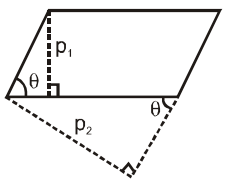
Thus any line parallel to ax + by + c = 0 is of the type ax + by + k = 0, where k is a parameter.

(iii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ &

$ax + by + c_2 = 0$ is $= \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$.

Note that coefficients of x & y in both the equations must be same.

(iv) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1-c_2)\,(d_1-d_2)}{m_1-m_2}\right|$.



Example # 19 : Find the equation of the straight line that has y-intercept 5 and is parallel to the straight line $3x - 7y = 8$.

Solution : Given line is $3x - 7y = 8$

$\therefore$ Slope of (1) is $\frac{3}{7}$

The required line is parallel to (1), so its slope is also 3/7, y-intercept of required line = 5

$\therefore$ By using $y = mx + c$ form, the equation of the required line is

$y = \frac{3}{7}x + 5$ or $3x - 7y + 35 = 0$

Example # 20 : Two sides of a square lie on the lines $5x - 12y + 6 = 0$ and $5x - 12y = 20$. What is its area ?

Solution : Clearly the length of the side of the square is equal to the distance between the parallel lines $5x - 12y + 6 = 0$.....(i) and $5x - 12y = 20$ .....(ii)

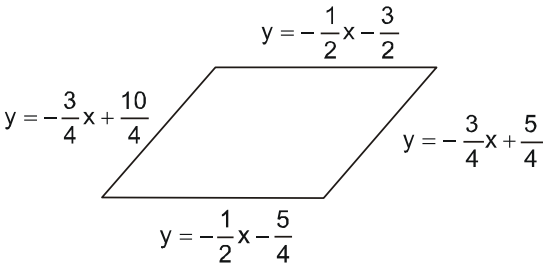
Now, Distance between the parallel lines

$$= \frac{|6 + 20|}{\sqrt{5^2 + (-12)^2}} = 2$$

Thus, the length of the side of the square is 2 and hence its area = 4

Example # 21 : Find the area of the parallelogram whose sides are $x + 2y + 3 = 0$, $3x + 4y - 5 = 0$, $2x + 4y + 5 = 0$ and $3x + 4y - 10 = 0$

Solution :



Here, $c_1 = -\frac{3}{2}$, $c_2 = -\frac{5}{4}$, $d_1 = \frac{10}{4}$, $d_2 = \frac{5}{4}$, $m_1 = -\frac{1}{2}$, $m_2 = -\frac{3}{4}$

$\therefore$ Area $= \left| \frac{\left(-\frac{3}{2} + \frac{5}{4}\right)\left(\frac{10}{4} - \frac{5}{4}\right)}{\left(-\frac{1}{2} + \frac{3}{4}\right)} \right| = \frac{5}{4}$ sq. units

Self practice problem :

(20) Find the area of parallelogram whose sides are given by $4x - 5y + 1 = 0$, $x - 3y - 6 = 0$, $4x - 5y - 2 = 0$ and $2x - 6y + 5 = 0$

Ans. (20) $\frac{51}{14}$ sq. units

Perpendicular Lines:

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slopes is -1, i.e. $m_1 m_2 = -1$. Thus any line perpendicular to $y = mx + c$ is of the form

$y = -\frac{1}{m}x + d$, where 'd' is any parameter.

(ii) Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where 'k' is any parameter.

Example # 22 : Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$

Solution : The equation of a line perpendicular to $3x + 2y + 5 = 0$ is

$2x - 3y + \lambda = 0$ .....(i)

This passes through the point (3, 4)

$\therefore$ $3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$

Putting $\lambda = 6$ in (i), we get $2x - 3y + 6 = 0$, which is the required equation.

Aliter The slope of the given line is –3/2. Since the required line is perpendicular to the given line. So, the slope of the required line is 2/3. As it passes through (3, 4). So, its equation is $y - 4 = \frac{2}{3}(x - 3)$

or $2x - 3y + 6 = 0$

Self practice problem :

(21) The vertices of a triangle are A(10, 4), B (–4, 9) and C(–2, –1). Find the equation of its altitudes. Also find its orthocentre.

Ans. (21) $x - 5y + 10 = 0$, $12x + 5y + 3 = 0$, $14x - 5y + 23 = 0$, $\left(-1, \frac{9}{5}\right)$

Position of the point (x_1, y_1) relative of the line $ax + by + c = 0$:

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non–origin side of $ax + by + c = 0$.

In general two points (x_1, y_1) and (x_2, y_2) will lie on same side or opposite side of $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

Example # 23 : Show that (2, –1) and (–3, 3) lie on the opposite sides of the line $2x - 3y + 5 = 0$.

Solution : At (2, –1), the value of $2x - 3y + 5 = 4 + 3 + 5 = 12 > 0$.

At (–3, 3), the value of $2x - 3y + 5 = -6 - 9 + 5 = -10 < 0$

$\therefore$ The points (2, –1) and (–3, 3) are on the opposite sides of the given line.

Self practice problems :

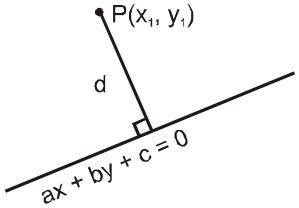
(22) Are the points (3, – 4) and (2, 6) on the same or opposite side of the line $3x - 4y = 8$?

(23) Which one of the points (1, 1), (–1, 2) and (2, 3) lies on the side of the line $4x + 3y - 5 = 0$ on which the origin lies?

Ans. (22) Opposite sides (23) (–1, 2)

Length of perpendicular from a point on a line :

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.



Example # 24 : Find the distance between the line $4x - 3y + 8 = 0$ and the point $(-2, 3)$

Solution : The required distance $= \left|\frac{(-2)\times 4 - 3\times 3 + 8}{\sqrt{4^2 + (-3)^2}}\right| = \frac{9}{5}$

Example # 25 : Find all points on $x - y + 2 = 0$ that lie at a unit distance from the line $12x - 5y + 9 = 0$.

Solution : Note that the co-ordinates of an arbitrary point on $x - y + 2 = 0$ can be obtained by putting $x = t$ (or $y = t$) and then obtaining y (or x) from the equation of the line, where t is a parameter. Putting $x = t$ in the equation $x - y + 2 = 0$ of the given line, we obtain $y = 2 + t$.
So, co-ordinates of an arbitrary point on the given line are $P(t, 2 + t)$. Let $P(t, 2 + t)$ be the required point. Then, distance of P from the line $12x - 5y + 9 = 0$ is unity i.e.

$\Rightarrow \quad \left|\frac{12t - 5(t+2) + 9}{\sqrt{12^2 + 5^2}}\right| = 1 \quad \Rightarrow \quad |7t - 1| = 13$

$\Rightarrow \quad 7t - 1 = \pm 13 \quad \Rightarrow \quad t = 2$ or $t = -12/7$

Hence, required points are (2,4) or (–12/7,2/7)

Self practice problem :

(24) Find the length of the altitudes from the vertices of the triangle with vertices : (–1, 1), (5, 2) and (3, –1).

Ans. (24) $\frac{16}{\sqrt{13}}, \frac{8}{\sqrt{5}}, \frac{16}{\sqrt{37}}$

Reflection of a point about a line :

(i) Foot of the perpendicular from a point (x_1, y_1) on the line $ax + by + c = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

(ii) The image of a point (x_1, y_1) about the line $ax + by + c = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

Example # 26 : Find the foot of perpendicular of the line drawn from P (2, –3) on the line $x - 2y + 5 = 0$.

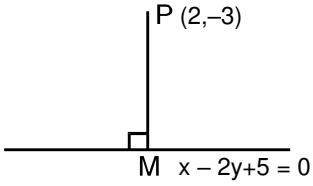
Solution : Slope of PM = – 2

$\therefore$ Equation of PM is

$2x + y = 1$ .....(i)

solving equation (i) with $x - 2y + 5 = 0$, we get

co-ordinates of M (–3/5, 11/5)



Aliter Here, $\frac{x-2}{1} = \frac{y+3}{-2} = -\frac{2+6+5}{1+4} \Rightarrow \frac{x-2}{1} = \frac{y+3}{-2} = -\frac{13}{5} \Rightarrow x = -3/5$, $y = 11/5$

Example # 27 : Find the image of the point P(–1, 2) in the line mirror $2x - 3y + 4 = 0$.

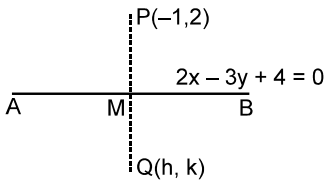
Solution : Let image of P is Q.

$\therefore$ PM = MQ & PQ $\perp$ AB

Let Q is (h, k)

$\therefore$ M is $\left(\frac{h-1}{2}, \frac{k+2}{2}\right)$

It lies on $2x - 3y + 4 = 0$.



Example # 28: Find the co-ordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 0), (5, 0) and (0, 12).

Solution : (i) We know that the co-ordinates of the centroid of a triangle whose angular points are $(x_1, y_1), (x_2, y_2) (x_3, y_3)$ are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

So the co-ordinates of the centroid of a triangle whose vertices are (0, 0), (5, 0) and (0, 12) are $\left(\frac{0+5+0}{3}, \frac{0+0+12}{3}\right)$ or $\left(\frac{5}{3}, 4\right)$.

(ii) Let A (0, 0), B (5, 0) and C(0, 12) be the vertices of triangle ABC.
Then c = AB = 5, b = CA =12
and a = BC = 13.

The co-ordinates of the in-centre are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

or $\left(\frac{13\times0+12\times5+5\times0}{5+12+13}, \frac{13\times0+12\times0+5\times12}{5+12+13}\right)$ or (2, 2)

Self practice problems :

(27) Two vertices of a triangle are (3, –5) and (–7, 4). If the centroid is (2, –1), find the third vertex.

(28) Find the co-ordinates of the centre of the circle inscribed in a triangle whose vertices are (– 36, 7), (20, 7) and (0, – 8)

Ans. (27) (10, – 2) (28) (–1, 0)

Bisectors of the angles between two lines:

Equations of the bisectors of angles between the lines ax + by + c = 0 & $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Example # 29 : Find the equations of the bisectors of the angle between the straight lines 3x + y + 1 = 0 and x + 3y + 1 = 0.

Solution : The equations of the bisectors of the angles between 3x+y+1= 0 and x+3y+1=0 are $\frac{3x+y+1}{\sqrt{3^2+1^2}}$

$= \pm\frac{x+3y+1}{\sqrt{1^2+3^2}}$ or 3x + y + 1 = ± (x + 3y + 1)

Taking the positive sign, we get x = y as one bisector
Taking the negative sign, we get 2x + 2y + 1 = 0 as the other bisector.

Self practice problem :

(29) Find the equations of the bisectors of the angles between the following pairs of straight lines 3x + 4y + 13 = 0 and 12x – 5y + 32 = 0

Ans. (29) 21x – 77y – 9 = 0 and 99x + 27y + 329 = 0

Methods to discriminate between the acute angle bisector & the obtuse angle bisector:

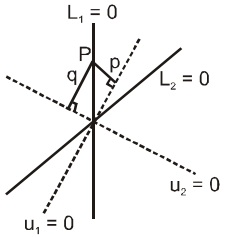
(i) If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.
If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.
If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(ii) Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown in figure. If,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.



(iii) If $aa' + bb' < 0$, then the equation of the bisector of this acute angle is

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

If, however, $aa' + bb' > 0$, the equation of the bisector of the obtuse angle is :

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

Example # 30 : For the straight lines $2x - y + 1 = 0$ and $x - 2y - 2 = 0$, find the equation of the
(i) bisector of the obtuse angle between them;
(ii) bisector of the acute angle between them;

Solution : $2x - y + 1 = 0$ .....(1)

and $x - 2y - 2 = 0$ .....(2)

Here $a_1 = 2, a_2 = 1, b_1 = -1, b_2 = -2$

Now $a_1a_2 + b_1b_2 = 4 > 0$

$\therefore$ bisector of the obtuse angle between lines (1) and (2) will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

or $x + y + 3 = 0$

and the equation of the bisector of the acute angle will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = -\frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

or $3x - 3y = 1$

Self practice problem :

(30) Find the equations of the bisectors of the angles between the lines $x + y - 3 = 0$ and $7x - y + 5 = 0$ and state which of them bisects the acute angle between the lines.

Ans. (30) $x - 3y + 10 = 0$ (bisector of the obtuse angle);
$6x + 2y - 5 = 0$ (bisector of the acute angle)

Condition of Concurrency :

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Alternatively : If three constants A, B & C (not all zero) can be found such that
$A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

Example # 31 : If the straight lines $x + 2y = 9$, $3x + 5y = 5$ and $ax - by = 1$ are concurrent. Then find the value of $35a + 22b$

Solution : Given lines are

$x + 2y = 9$ .....(1)

$3x + 5y = 5$ .....(2)

and $ax - by = 1$ .....(3)

Lines will be concurrent if $\Delta = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & -b & -1 \end{vmatrix} = 0 \Rightarrow 35a + 22b = -1$$

Self practice problem :

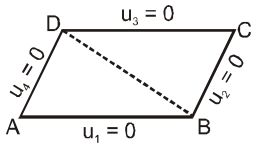
(31) Find the value of m so that the lines $4x - 3y + 2 = 0$, $3x + 4y - 4 = 0$ and $x + my + 6 = 0$ may be concurrent.

Ans. (31) –7

Family of Straight Lines :

The equation of a family of straight lines passing through the point of intersection of the lines, $L_1 \equiv a_1x + b_1y + c_1 = 0$ & $L_2 \equiv a_2x + b_2y + c_2 = 0$ is given by $L_1 + k L_2 = 0$ i.e. $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note : (i) If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$ then $u_1=0$; $u_2= 0$; $u_3= 0$; $u_4= 0$ form a parallelogram. The diagonal BD can be given by $u_2u_3 - u_1u_4 = 0$.



(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some real λ and μ.

[For getting the values of λ & μ compare the coefficients of x, y & the constant terms].

Example # 32 : If $3a + 2b + 5c = 0$ and the set of lines $ax + by + c = 0$ passes through a fixed point . Find co-ordinates of that point.

Solution : $3a + 2b + 5c = 0$ ..... (i)

$ax + by + c = 0$ ..... (ii)

Eliminating c, we get.

$$ax + by - \frac{1}{5}(3a + 2b) = 0 \Rightarrow a\left(x - \frac{3}{5}\right) + b\left(y - \frac{2}{5}\right) = 0 \Rightarrow \left(x - \frac{3}{5}\right) + \frac{b}{a}\left(y - \frac{2}{5}\right) = 0$$

It is of the form $L_1 + \lambda L_2 = 0$

Which passes through the point of intersection $\left(\frac{3}{5}, \frac{2}{5}\right)$ of $L_1= 0$ & $L_2= 0$ for all real values of a & b

Aliter : $3a + 2b + 5c = 0 \quad \Rightarrow \quad \frac{3}{5}a + \frac{2}{5}b + c = 0$

$\left(\frac{3}{5}, \frac{2}{5}\right)$ lies on the line $ax + by + c = 0$ Hence fixed point $\left(\frac{3}{5}, \frac{2}{5}\right)$

Example # 33 : Obtain the equations of the lines passing through the intersection of lines $3x + 7y = 17$ and $x + 2y = 5$ and is prependicular to the straight line $3x + 4y = 10$.

Solution : The equation of any line through the intersection of the given lines is

$(x + 2y – 5) + \lambda (3x + 7y – 17) = 0$

or $x (3 \lambda + 1) + y (7\lambda + 2) – 17\lambda – 5 = 0$ .....(i)

This is prependicular to the line $3x + 4y = 10$

$$\therefore \left(-\frac{3\lambda+1}{7\lambda+2}\right)\left(-\frac{3}{4}\right) = -1 \Rightarrow \lambda = -\frac{11}{37}$$

Putting this value of λ in (i), the equation of required line $4x – 3y + 2 = 0$

Self practice problem :

(32) Find the equation of the lines through the point of intersection of the lines $x – 3y + 1 = 0$ and $2x + 5y – 9 = 0$ and whose distance from the origin is $\sqrt{5}$

Ans. (32) $2x + y – 5 = 0$

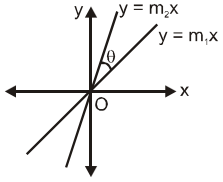
A Pair of straight lines through origin:

(i) A homogeneous equation of degree two, "$ax^2 + 2hxy + by^2 = 0$" always represents a pair of straight lines passing through the origin if :

(a) $h^2 > ab$ $\Rightarrow$ lines are real & distinct .

(b) $h^2 = ab$ $\Rightarrow$ lines are coincident .

(c) $h^2 < ab$ $\Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)



This equation is obtained by multiplying the two equations of lines

$(m_1x – y) (m_2x – y) = 0$

$\Rightarrow m_1m_2x^2 – (m_1 + m_2) xy + y^2 = 0$

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$$m_1 + m_2 = -\frac{2h}{b} \text{ \& } m_1 m_2 = \frac{a}{b}.$$

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$.

(iv) The condition that these lines are :

(a) at right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 +co–efficient of $y^2 = 0$.

(b) coincident is $h^2 = ab$.

(c) equally inclined to the axis of x is $h = 0$.i.e. coeff. of $xy = 0$.

Note that a homogeneous equation of degree n represents n straight lines passing through origin.

Example # 34 : Show that the equation $18x^2 - 9xy + y^2 = 0$ represents a pair of distinct straight lines, each passing through the origin. Find the separate equations of these lines.

Solution : The given equation is a homogeneous equation of second degree. So, it represents a pair of straight lines passing through the origin. Comparing the given equation with $ax^2 + 2hxy + by^2 = 0$, we obtain $a = 18$, $b = 1$ and $2h = -9$.

$\therefore \quad h^2 - ab = \frac{81}{4} - 18 = \frac{9}{4} > 0 \quad \Rightarrow \quad h^2 > ab$

Hence, the given equation represents a pair of distinct lines passing through the origin.

Now, $18x^2 - 9xy + y^2 = 0 \Rightarrow \left(\frac{y}{x}\right)^2 - 9\left(\frac{y}{x}\right) + 18 = 0 \Rightarrow \left(\frac{y}{x} - 6\right)\left(\frac{y}{x} - 3\right) = 0$

$\Rightarrow \quad -6 = 0$ or $-3 = 0 \Rightarrow y - 6x = 0$ or $y - 3x = 0$

So the given equation represents the straight lines $y - 3x = 0$ and $y - 6x = 0$.

Example # 35 : Find the equations to the pair of lines through the origin which are perpendicular to the lines represented by $6x^2 - xy - 12y^2 = 0$.

Solution : We have $6x^2 - xy - 12y^2 = 0$.

$\Rightarrow \quad 6x^2 - 9xy + 8xy - 12y^2 = 0 \quad \Rightarrow \quad (3x + 4y)(2x - 3y) = 0$

$\Rightarrow \quad 3x + 4y = 0$ & $2x - 3y = 0$

Thus the given equation represents the lines $3x + 4y = 0$ and $2x - 3y = 0$. The equations of the lines passing through the origin and perpendicular to the given lines are $4x - 3y = 0$ & $3x + 2y = 0$ there combined equations $12x^2 - xy - 6y^2 = 0$

Example # 36 : Find the angle between the pair of straight lines $x^2 - 3xy + 2y^2 = 0$

Solution : Given equation is $x^2 - 3xy + 2y^2 = 0$

Here a = coeff. of $x^2 = 1$, b = coeff. of $y^2 = 2$

and $2h$ = coeff. of $xy = -3 \quad \therefore \quad h = -\frac{3}{2}$

Now $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \left|\frac{2\sqrt{\frac{9}{4} - 2}}{1 + 2}\right| = \frac{1}{3}$

Where θ is the acute angle between the lines.

$\therefore$ acute angle between the lines is $\tan^{-1}\left(\frac{1}{3}\right)$ and obtuse angle between them is

$\pi - \tan^{-1}\left(\frac{1}{3}\right)$

Self practice problems :

(33) Find the area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 9$.

Ans. (33) $\frac{27}{4}$ sq. units

General equation of second degree representing a pair of Straight lines :

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

Such an equation is obtained again by multiplying the two equation of lines $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

Example # 38 : Prove that the equation $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$ represents a pair of straight lines. Find the co-ordinates of their point of intersection.

Solution : Given equation is $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$

Writing the equation (1) as a quadratic equation in x we have

$6x^2 + (13y + 8)\,x + 6y^2 + 7y + 2 = 0$

$$\therefore \quad x = \frac{-(13y+8) \pm \sqrt{(13y+8)^2 - 4.6(6y^2+7y+2)}}{12}$$

$$= \frac{-(13y+8) \pm \sqrt{169y^2 + 208y + 64 - 144y^2 - 168y - 48}}{12}$$

$$= \frac{-(13y+8) \pm \sqrt{25y^2 + 40y + 16}}{12} = \frac{-(13y+8) \pm (5y+4)}{12}$$

$\Rightarrow$ $12x = -13y - 8 + 5y + 4$ or $12x = -13y - 8 - 5y - 4$

$\Rightarrow$ $3x + 2y + 1 = 0$ or $2x + 3y + 2 = 0$

Hence equation (1) represents a pair of straight lines whose equation are

$3x + 2y + 1 = 0$ .....(1) and $2x + 3y + 2 = 0$ .....(2)

Solving these two equations, the required point of intersection is

Self practice problem :

(35) Find the combined equation of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ and find the angle between them.

Ans. (35) $x^2 - 5xy + 4y^2 + 3x - 3y = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

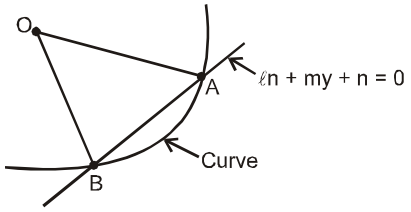
Homogenization :

This method is used to write the joint equation of two lines connecting origin to the points of intersection of a given line and a given second degree curve. The equation of a pair of straight lines joining origin to the points of intersection of the line $L \equiv \ell x + my + n = 0$ and a second degree curve

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{\ell x + my}{-n}\right) + 2fy\left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right)^2 = 0.$

The equation is obtained by homogenizing the equation of curve with the help of equation of line.



Notes : (i) Here we have written 1 as $\frac{\ell x + my}{-n}$ and converted all terms of the curve to second degree expressions

(ii) Equation of any curve passing through the points of intersection of two curves $C_1 = 0$ and $C_2 = 0$ is given by $\lambda C_1 + \mu C_2 = 0$, where λ & μ are parameters.

Example # 39 : All chords of the curve $2x^2 - 3y^2 + 4x - 2y = 0$ which subtend a right angle at the origin passes through a fixed point. Find that point

Solution : Let $Ax + By = 1$ be a chord of the curve $2x^2 - 3y^2 + 4x - 2y = 0$

Equation of the given curve is $2x^2 - 3y^2 + 4x - 2y = 0$

and equation of the chord is $Ax + By = 1$ ....(i)

Making equation (1) homogeneous equation of the second degree in x any y with the help of (1), we have $2x^2 - 3y^2 + 4x(Ax + By) - 2y(Ax + By) = 0$

$x^2(2 + 4A) + y^2(-3 - 2B) + xy(4B - 2A) = 0$ ....(ii)

since line represented by (ii) are at right angle

$\Rightarrow$ coefficient of x^2+ coefficient of $y^2 = 0$

$\Rightarrow$ $2 + 4A + (-3 - 2B) = 0$

$4A - 2B = 1$

chord $Ax + By = 1$

$x = 4 \quad y = -2$

fixed point $(4, -2)$

Self practice problems :

(36) Find the equation of the straight lines joining the origin to the points of intersection of the line $3x + 4y - 5 = 0$ and the curve $2x^2 + 3y^2 = 5$.

(37) Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

Ans. (36) $x^2 - y^2 - 24xy = 0$ (37) $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

A-1. (i) Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral triangle whose side is 2a.

(ii) Find the points which trisect the line segment joining the points (0, 0) and (9, 12).

A-2. (i) In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (– 7, 9)?

(ii) In which ratio P(2a – 2, 4a – 6)) divides Q (2a – 3, 3a – 7) and R(2a, 6a – 4).

A-3. (i) Find the value of λ such that points P(1,2), Q(–2, 3) and R(λ + 1, λ) are not forming a triangle ?

(ii) Find the ratio in which the line segment joining of the points (1, 2) and (– 2, 3) is divided by the line 3x + 4y = 7

(iii) Find the harmonic conjugate of the point R (5, 1) with respect to points P (2, 10) and Q (6, – 2).

A-4. A and B are the points (3, 4) and (5, – 2) respectively. Find the co-ordinates of a point P such that PA = PB and the area of the triangle PAB = 10.

A-5. Find the area of the quadrilateral with vertices as the points given in each of the following :

(i) (0, 0), (4, 3), (6, 0), (0, 3) (ii) (0, 0), (a, 0), (a, b), (0, b)

Section (B) : Different forms of straight lines and Angle between lines

B-1. Reduce $x + \sqrt{3}\ y + 4 = 0$ to the :

(i) Slope intercepts form and find its slope and y-intercept.

(ii) Intercepts form and find its intercepts on the axes.

(iii) Normal form and find values of P and α.

B-2. Find number of straight line passing through (2, 4) and forming a triangle of 16 sq. cm with the coordinate axis.

B-3. Find the equation of the straight line that passes through the point A(– 5, – 4) and is such that the portion intercepted between the axes is divided by the point A in the ratio 1 : 2 (internally).

B-4. The co-ordinates of the mid-points of the sides of a triangle are (2, 1), (5, 3) and (3, 7). Find the length and equation of its sides.

B-5. Find the straight line cutting an intercept of one unit on negative x-axis and inclined at 45° (in anticlockwise direction) with positive direction of x-axis

B-6. Through the point P(4, 1) a line is drawn to meet the line 3x – y = 0 at Q where $PQ = \frac{11}{2\sqrt{2}}$. Determine the equation of line.

B-7. A line having slope '1' is drawn from a point A(–3, 0) cuts a curve $y = x^2 + x + 1$ at P & Q. Find ℓ(AP) ℓ(AQ).

B-8. Find the direction in which a straight line must be drawn through the point (1, 2), so that its point of intersection with the line $x + y = 4$ may be at a distance $\frac{\sqrt{6}}{3}$ from this point.

B-9. Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line $x - y = 2$. Find their equations and also find the area of triangle bounded by the three lines.

B-10. Find the equation of a straight lines which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line $2x + 3y + 4 = 0$

B-11. From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. Find co-ordinates of point Q.

B-12. The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6), then find the locus of third vertex of triangle.

B-13. A point moves in the x-y plane such that the sum of its distances from two mutually perpendicular lines is always equal to 3, then find the area enclosed by the locus of the point.

B-14. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Then find the equations of other sides.

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot, Area of Parallelogram

C-1. Plot the region
(i) $6x + 2y \geq 31$ (ii) $2x + 5y \leq 10$ (iii) $8x + 3y + 6 > 0$ (iv) $x > 2$

C-2. Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line $y = 3x - 4$.

C-3. Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point A(4, 8) and reflected line passes through the point B (8, 12). Compute the slope of the mirror.

C-4. Find the nearest point on the line $3x + 4y - 1 = 0$ from the origin.

C-5. Find the position of the origin with respect to the triangle whose sides are $x + 1 = 0$, $3x - 4y - 5 = 0$, $5x + 12y - 27 = 0$.

C-6. Find the area of parallelogram whose two sides are $y = x + 3$, $2x - y + 1 = 0$ and remaining two sides are passing through (0, 0).

C-7. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so, find λ.

C-8. Find the equations of two straight lines which are parallel to $x + 7y + 2 = 0$ and at $\sqrt{2}$ distance away from it.

C-9. Prove that the area of the parallelogram contained by the lines $4y - 3x - a = 0$, $3y - 4x + a = 0$, $4y - 3x - 3a = 0$ and $3y - 4x + 2a = 0$ is $\frac{2}{7}a^2$.

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

D-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of
(i) Centroid (ii) Circumcentre

(iii) Incentre (iv) Excentre opposite to vertex (5, 12)

D-2. Find the sum of coordinates of the orthocentre of the triangle whose sides are x = 3, y = 4 and 3x + 4y = 6.

D-3. Find equations of altitudes and the co-ordinates of the othocentre of the triangle whose sides are 3x – 2y = 6, 3x + 4y + 12 = 0 and 3x – 8y + 12 = 0.

D-4. A triangle has the lines $y = m_1x$ and $y = m_2x$ for two of its sides, where m_1, m_2 are the roots of the equation $x^2 + ax - 1 = 0$, then find the orthocentre of triangle.

D-5. Prove that the circumcentre, orthocentre, incentre & centroid of the triangle formed by the points A(–1, 11); B (–9, –8); C (15, –2) are collinear, without actually finding any of them.

D-6. Find locus of centroid of ΔAOB if line AB passes through (3, 2), A and B are on coordinate axes.

D-7. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1, 0), where 't' is the parameter.

D-8. Show that equation of the locus of a point which moves so that difference of its distance from two given points (ae, 0) and (–ae, 0) is equal to 2a is $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$.

D-9. Find the locus of point of intersection of the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$, where α is a parameter.

D-10. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P.

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

E-1. Find equations of acute and obtuse angle bisectors of the angle between the lines 4x + 3y – 7 = 0 and 24x + 7y – 31 = 0.

E-2. Find the equation of a straight line passing through the point (4, 5) and equally inclined to the lines 3x = 4y + 7 and 5y = 12x + 6.

E-3. The line x + 3y – 2 = 0 bisects the angle between a pair of straight lines of which one has equation x – 7y + 5 = 0,then find equation of other line

E-4. Find the value of λ such that lines x + 2y = 3, 3x – y = 1 and $\lambda x + y = 2$ can not form a triangle.

E-5. Find values of λ for which line y = x + 1, $y = \lambda x + 2$ and $y = (\lambda^2 + \lambda - 1)x + 3$ are con-current.

E-6. Find the equation to the straight line passing through
(i) The point (3, 2) and the point of intersection of the lines 2x + 3y = 1 and 3x – 4y = 6.
(ii) The intersection of the lines x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the straight line y – x = 8.

E-7. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines ax + by + c = 0 and $\ell x + my + n = 0$.

Section (F) : Pair of straight lines, Homogenization

F-1. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the n^{th} power of the other, then prove that $(ab^n)^{\frac{1}{n+1}} + (a^nb)^{\frac{1}{n+1}} + 2h = 0$

F-2. For what value of λ does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represent a pair of straight lines ? Find their
(i) equations, (ii) point of intersection, (iii) acute angle between them.

F-3. (i) Find the integral values of 'h' for which $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ represents two real straight lines.
(ii) If the pair of lines represented by equation $k(k - 3)x^2 + 16xy + (k + 1)y^2 = 0$ are perpendicular to each other, then find k.

F-4. Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

F-5. Find the condition that the diagonals of the parallelogram formed by the lines $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$, $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

A-1. Mid point of A(0, 0) and B(1024, 2048) is A_1. mid point of A_1 and B is A_2 and so on. Coordinates of A_{10} are.
(A) (1022, 2044) (B) (1025, 2050) (C) (1023, 2046) (D) (1, 2)

A-2. If the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear, the number of possible values of k are
(A) 4 (B) 2 (C) 1 (D) 3

A-3. Given a ΔABC with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals :
(A) 1 (B) 2 (C) 3 (D) Infinite

A-4. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.
Statement-1: MP, MB, MQ are in G.P.
Statement-2 AP, AB and AQ are in HP.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for
STATEMENT-1Pro
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

A-5. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3)
(A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

Section (B) : Different forms of straight lines and Angle between lines

B-1. A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is :
(A) $x + 2y = 5$ (B) $x - y + 1 = 0$ (C) $x + y - 3 = 0$ (D) $2x + y - 4 = 0$

B-2. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is
(A) 133 (B) 190 (C) 233 (D) 105

B-3. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15º. The equation of the line in the new position is :
(A) $x - \sqrt{3}\,y - 2 = 0$ (B) $x - 2y - 2 = 0$ (C) $\sqrt{3}\,x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

B-4. In a ΔABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation $x + 2y = 16$. If the mid point of BC is (5, 6), then the equation of BC is
(A) $2x + y = 16$ (B) $x + y = 11$ (C) $2x - y = 4$ (D) $x + y = 10$

B-5. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is :
(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ (B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ (D) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

B-6. Find equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120º with the positive direction of x-axis.
(A) $\sqrt{3}\,x - y = 0$ (B) $\sqrt{3}\,x + y = 8$ (C) $x + \sqrt{3}\,y = 8$ (D) $x - \sqrt{3}\,y = 8$

B-7. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :
(A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

B-8. If a point P(x, y) from where line drawn cuts coordinate axes at A and B (with A on x-axis and B on y-axis) satisfies $\alpha.\frac{x^2}{PB^2} + \beta\frac{y^2}{PA^2} = 1$, then $\alpha + \beta$ is
(A) 1 (B) 2 (C) 3 (D) 4

B-9. Two particles start from the point (2, – 1), one moving 2 units along the line $x + y = 1$ and the other 5 units along the line $x - 2y = 4$. If the particles move towards increasing y, then their new positions are
(A) $\left(2-\sqrt{2}, \sqrt{2}-1\right)$, $\left(2\sqrt{5}+2, \sqrt{5}-1\right)$ (B) $\left(2\sqrt{5}+2, \sqrt{5}-1\right)$, $\left(2\sqrt{2}, \sqrt{2}+1\right)$
(C) $\left(2+\sqrt{2}, \sqrt{2}+1\right)$, $\left(2\sqrt{5}+2, \sqrt{5}+1\right)$ (D) none of these

B-10. Equation of a straight line passing through the origin and making with x – axis an acute angle twice the size of the angle made by the line $y = (0.2)x$ with the x – axis, is :
(A) $y = (0.4)x$ (B) $y = (5/12)x$ (C) $6y - 5x = 0$ (D) $6y + 5x = 0$

B-11. The points A (1, 3) and C (5,1) are the oppositive vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is
(A) $2x+y-8 = 0$ (B) $2x-y-4 = 0$ (C) $2x-y+4 = 0$ (D) $2x+y = 0$

B-12. The point (–4,5) is the vertex of a square and one of its digonals is $7x-y+8 = 0$. The equation of the other diagonal is
(A) $7x-y+23 = 0$ (B) $7y + x = 30$ (C) $7y + x = 31$ (D) $x - 7y = 30$

B-13. Two straight lines $x + 2y = 2$ and $x + 2y = 6$ are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally
(A) $3x + 6y + 8 = 0$ (B) $3x + 6y = 14$ (C) $3x + 6y + 14 = 0$ (D) $3x + 2y = 10$

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot

C-1. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0 \;\forall\; a \in R, b > 0$ are :
(A) $b \in (2, 4)$ (B) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

C-2. The point $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin, then
(A) $a \geq 1$ or $a \leq -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$
(C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$

C-3. Find area of region represented by $3x + 4y > 12$, $4x + 3y > 12$ and $x + y < 4$
(A) $\frac{8}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$

C-4. The image of the point A (1, 2) by the line mirror $y = x$ is the point B and the image of B by the line mirror $y = 0$ is the point (α, β), then :
(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$

C-5. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is $(1, -2)$, then the equation of the line BC is :
(A) $14x + 23y = 40$ (B) $14x - 23y = 40$ (C) $23x + 14y = 40$ (D) $23x - 14y = 40$

C-6. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is $x + 3y - \lambda = 0$, then the value of λ is
(A) 11 (B) 12 (C) 13 (D) 14

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

D-1. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin, then the co–ordinates of C are :
(A) (2a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2 + b^2}, 0\right)$ (D) none

D-2. A triangle ABC with vertices A $(-1, 0)$, B $(-2, 3/4)$ & C $(-3, -7/6)$ has its orthocentre H. Then the orthocentre of triangle BCH will be :
(A) $(-3, -2)$ (B) (1, 3) (C) $(-1, 2)$ (D) none of these

D-3. Find locus of centroid of ΔABC, if B(1, 1), C(4, 2) and A lies on the line $y = x + 3$.
(A) $3x + 3y + 1 = 0$ (B) $x + y = 3$ (C) $3x - 3y + 1 = 0$ (D) $x - y = 3$

D-4. The locus of the mid-point of the distance between the axes of the variable line $x \cos \alpha + y \sin \alpha = p$, where p is constant, is
(A) $x^2 + y^2 = 4p^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{2}{p^2}$

D-5. Find the locus of a point which moves so that sum of the squares of its distance from the axes is equal to 3.
(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = 3$ (C) $|x| + |y| = 3$ (D) $x^2 - y^2 = 3$

D-6. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is :
(A) $bx + ay - 3xy = 0$ (B) $bx + ay - 2xy = 0$ (C) $ax + by - 3xy = 0$ (D) $ax + by - 2xy = 0$

D-7. Consider a triangle ABC, whose vertices are A(–2, 1), B(1, 3) and C(x, y). If C is a moving point such that area of ΔABC is constant, then locus of C is :
(A) Straight line (B) Circle (C) Ray (D) Parabola

D-8. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)\,x + (b_1 - b_2)\,y + c = 0$, then the value of 'c' is :

(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

E-1. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through $(-11, 4)$, the equation of acute angle bisector of L_1 & L_2 is :

(A) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$ (C) $2x + 16y + 5 = 0$ (D) $2x + 16y - 5 = 0$

E-2. The equation of the internal bisector of $\angle BAC$ of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is

(A) $2x + y + 12 = 0$ (B) $x + 2y - 12 = 0$ (C) $2x + y - 12 = 0$ (D) $2x - y - 12 = 0$

E-3. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the point $(-1, 4)$ is :

(A) $21x + 27y - 121 = 0$ (B) $21x - 27y + 121 = 0$

(C) $21x + 27y + 191 = 0$ (D) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

E-4. The least positive value of t so that the lines $x = t + a$, $y + 16 = 0$ and $y = ax$ (where a is real variable) are concurrent is

(A) 2 (B) 4 (C) 16 (D) 8

E-5. Consider the family of lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -

(A) $25x - 62y + 86 = 0$ (B) $62x - 25y + 86 = 0$ (C) $25x - 62y = 86$ (D) $5x - 2y - 7 = 0$

E-6. The equation of a line of the system $2x + y + 4 + \lambda(x - 2y - 3) = 0$ which is at a distance $\sqrt{10}$ units from point A(2, –3) is

(A) $3x + y + 1 = 0$ (B) $3x - y + 1 = 0$ (C) $y - 3x + 1 = 0$ (D) $y + 3x - 2 = 0$

E-7. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, are concurrent at the point

(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

E-8. Find the equation of a straight line which passes through the point of intersection of the straight lines $x + y - 5 = 0$ and $x - y + 3 = 0$ and perpendicular to the straight line intersecting x-axis at the point (–2, 0) and the y-axis at the point (0, –3),

(A) $2x + 3y + 10 = 0$ (B) $2x - 3y + 10 = 0$ (C) $2x - 5y + 10 = 0$ (D) $2x + 5y + 10 = 0$

E-9. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(A) Above the x-axis at a distance of 3/2 from it (B) Above the x-axis at a distance of 2/3 from it

(C) Below the x-axis at a distance of 3/2 from it (D) Below the x-axis at a distance of 2/3 from it

Section (F) : Pair of straight lines, Homogenization

F-1. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =

(A) 1 (B) 2 (C) 4 (D) 16

F-2. The equation of second degree $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$ represents a pair of straight lines. The distance between them is

(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $2\sqrt{3}$

F-3. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

PART - III : MATCH THE COLUMN

1.

	Column –I		**Column – II**
(A)	The points A(–4, –1), B (–2, –4), C (4, 0) and D(2, 3) are the vertices of	(p)	Square
(B)	The figure formed by the lines $2x + 5y + 4 = 0$, $5x + 2y + 7 = 0$, $2x + 5y + 3 = 0$ and $5x + 2y + 6 = 0$ is	(q)	Rectangle
(C)	A quadrilateral whose diagonal are perpendicular bisectors of each other must be	(r)	Rhombus
(D)	A quadrilateral whose diagonals are angle bisector of sides and bisect each other must be	(s)	Parallelogram

2. **Match The Followings :**

	Column –I		**Column – II**
(A)	P lies on the line $y = x$ and Q lies on $y = 2x$. The equation for the locus of the mid point of PQ, if $\|PQ\| = 4$, is $25x^2 - \lambda xy + 13y^2 = 4$, then λ equals	(p)	36
(B)	The line $(K + 1)^2 x + ky - 2K^2 - 2 = 0$ passes through a point (α, β) regardless of value K. Then $(\alpha - \beta)$ equals :	(q)	6
(C)	Let two lines be $C_1 : 3x - 4y + 1 = 0$ and $C_2 : 8x + 6y + 1 = 0$ suppose (α, β) is a point which is at unit distance from each of the given lines. then sum of all possible value of α is	(r)	–4/5
(D)	If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then 'a/5' is equal to	(s)	– 1

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. On the portion of the straight line $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co–ordinates
(A) (2, 3) (B) (3, 2) (C) (3, 3) (D) (2, 2)

2. If the straight line $ax + by + p = 0$ and $x\cos\alpha + y\sin\alpha = p$ enclosed an angle of $\frac{\pi}{4}$ and the line $x\sin\alpha - y\cos\alpha = 0$ meets them at the same point, then $a^2 + b^2$ is
(A) 4 (B) 3 (C) 2 (D) 1

3. A ΔABC is formed by the lines $2x - 3y - 6 = 0$, $3x - y + 3 = 0$ and $3x + 4y - 12 = 0$. If the points $P(\alpha, 0)$ and $Q(0, \beta)$ always lie on or inside the ΔABC, then ;

(A) $\alpha \in [-1, 2]$ & $\beta \in [-2, 3]$ (B) $\alpha \in [-1, 3]$ & $\beta \in [-2, 4]$

(C) $\alpha \in [-2, 4]$ & $\beta \in [-3, 4]$ (D) $\alpha \in [-1, 3]$ & $\beta \in [-2, 3]$

4. If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines $x + 2y = 1$ and $2x + 4y = 15$ is

(A) $-\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$ (B) $0 < t < \frac{5\sqrt{2}}{6}$ (C) $-\frac{4\sqrt{2}}{5} < t < 0$ (D) $-\frac{4\sqrt{2}}{3} < t < \frac{\sqrt{2}}{6}$

5. The point A(4, 1) undergoes following transformations successively :

(i) reflection about line $y = x$

(ii) translation through a distance of 3 units in the positive direction of x-axis

(iii) rotation through an angle 105° in anti-clockwise direction about origin O.

Then the final position of point A is

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

6. Given two points $A \equiv (-2, 0)$ and $B \equiv (0, 4)$, then find coordinate of a point P lying on the line $2x - 3y = 9$ so that perimeter of Δ APB is least.

(A) $\left(\frac{42}{13}, -\frac{11}{3}\right)$ (B) $\left(\frac{84}{13}, -\frac{74}{13}\right)$ (C) $\left(\frac{21}{17}, -\frac{37}{17}\right)$ (D) $(0, -3)$

7. A ray of light is sent from the point (1, 4). Upon reaching the x-axis, the ray is reflected from the point (3, 0). This reflected ray is again reflected by the line $x + y = 5$ and intersect y-axis at P. Find the co-ordinate of P.

(A) $\left(\frac{1}{2}, 0\right)$ (B) $\left(0, \frac{-1}{2}\right)$ (C) $\left(0, \frac{1}{3}\right)$ (D) $\left(2, \frac{1}{-2}\right)$

8. AB is a variable line sliding between the co-ordinate axes in such a way that A lies on X-axis and B lies on Y-axis. If P is a variable point on AB such that PA = b, PB = a and AB = a + b, then equation of locus of P is

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) $x^2 - y^2 = a^2 + b^2$

9. If the distance of any point (x, y) from the origin is defined as $d(x, y) = \max \{ |x|, |y| \}$, $d(x, y) = a$ (where 'a' is non-zero constant), then the locus is

(A) A circle (B) Straight line (C) A square (D) A triangle

10. Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed. Then find locus of the foot of the perpendicular drawn from P to AB.

(A) $x^{2/3} + y^{2/3} = c^{2/3}$ (B) $x^{2/3} + y^{2/3} = c^{1/3}$ (C) $x^{1/3} + y^{1/3} = c^{2/3}$ (D) $x^{1/3} + y^{1/3} = c^{1/3}$

11. Let the line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the x and y axes at A and B respectively. Now a line parallel to the given line cuts the coordinate axis at P and Q and points P and Q are joined to B and A respectively. The locus of intersection of the joining lines is

(A) $\frac{x}{a} - \frac{y}{b} = 0$ (B) $\frac{x}{a} + \frac{y}{b} = 0$ (C) $\frac{x}{b} - \frac{y}{a} = 0$ (D) $\frac{x}{b} + \frac{y}{a} = 0$

12. A variable line whose slope is –2 cuts the x and y axes respectively at points A and C. A rhombus ABCD is completed such that vertex B lies on the line y = x. Then the locus of vertex D is
(A) $2x + y = 1$ (B) $x – y = 0$ (C) $x + y = 0$ (D) $x + 2y = 0$

13. ABCD is a square away from origin of side length 'a'. Its side AB slides between x and y–axes in first quadrant with A on x-axis and B on y-axis. The locus of the foot of perpendicular dropped from the point E on the diagonal AC (where E is the midpoint of the side AD), is

(A) $(y – x)^2 + (x – 3y)^2 = a^2$ (B) $(y – x)^2 + (x – 3y)^2 = \frac{a^2}{2}$

(C) $(y – x)^2 + (x – 3y)^2 = \frac{a^2}{4}$ (D) None of these

14_. Let orthocentre of ΔABC is (4, 6). If A = (4, 7) and B = (–2, 4), the find coordinates of vertex C
(A) (5, 4) (B) (4, 5) (C) (–5, –4) (D) (–4, –5)

15_. Let Incentre of ΔABC is (2, 5). If A = (1, 13) and B = (–4, 1), then find coordinates of C
(A) (1, 10) (B) (10, 1) (C) (8, 2) (D) (9, 3)

16. The locus of circumcentre of the triangle formed by vertices A((–pq – p – q), –(1 + p)(1 + q)), B(pq + p – q, (1 + p)(1 + q)), C(pq + q – p, (1 + p)(1 + q)) is
(A) $y + x = 0$ (B) $y – x = 0$ (C) $x^2 + y^2 = 1$ (D) $xy = 1$

17. Let two sides of rectangle of area 20 units are along lines $x – y = 0$ and $x + y = 2$, then the locus of point of intersection of diagnals is
(A) $(x –1)^2 + (y –1)^2 = 10$ or $(y –1)^2 + (x –1)^2 = 10$
(B) $(x –1)^2 – (y –1)^2 = 10$ or $(y –1)^2 – (x –1)^2 = 10$
(C) $(x +1)^2 – (y +1)^2 = 10$ or $(y +1)^2 – (x +1)^2 = 10$
(D) $(x –1)^2 + (y –1)^2 = 10$ or $(y +1)^2 – (x +1)^2 = 10$

18. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 – y^2 + 2y = 1$ is
(A) 2 sq units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units

19. Equation of the line pair through the origin and perpendicular to the line pair $xy – 3y^2 + y – 2x + 10 = 0$ is
(A) $xy – 3y^2 = 0$ (B) $xy + 3x^2 = 0$ (C) $xy + 3y^2 = 0$ (D) $x^2 – y^2 = 0$

20. Find the equation of the two straight lines which together with those given by the equation $6x^2 – xy – y^2 + x + 12y – 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.
(A) $6x^2 – xy – y^2 – x – 12y – 35 = 0$ (B) $6x^2 – xy – y^2 – x – 12y + 35 = 0$
(C) $6x^2 – xy – y^2 – x + 12y – 35 = 0$ (D) $6x^2 – xy – y^2 + x – 12y – 35 = 0$

21. The curve passing through the points of intersection of $S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} – 1 = 0$ and

$S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines which are
(A) equally inclined to the x - axis (B) perpendicular to each other
(C) parallel to each other (D) Not equally inclined to y-axis

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc– (ab + bc + ac) + 3 (a + b + c).

2. The complete set of values of λ for which point $(\lambda, \lambda + 1)$ is an interior points of ΔABC, where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$ is (p,q) then value of $p^2 + q^2$ is

3. Let ABC be a triangle such that the coordinates of the vertex A are (– 3, 1). Equation of the median through B is 2x + y – 3 = 0 and equation of the angular bisector of C is 7x – 4y – 1 = 0. Find the slope of line BC.

4. A(3, 4), B(0, 0) and C(3, 0) are vertices of ΔABC. If 'P' is the point inside the ΔABC, such that $d(P, BC) \leq$ min. {d(P, AB), d (P, AC)}. Then the maximum of d (P, BC) is.
(where d(P, BC) represent distance between P and BC).

5. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line 2x + y = 13. Then find the area of the triangle.

6. On the straight line y = x + 2, a point (a, b) is such that the sum of the square of distances from the straight lines 3x – 4y + 8 = 0 and 3x – y – 1 = 0 is least, then find value of (a + b).

7. Parallelogram ABCD is cut by (2n –1) number of parallel lines in which one is diagnal AC. Distance between any two nearest lines is same which is also equal to distance of B, D from respective nearest line anong these. Ratio of area of smallest triangle so formed to area of parallelogram is $\frac{1}{32}$. Find n.

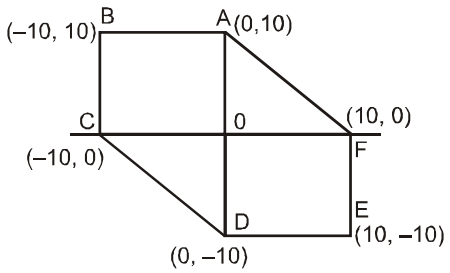
8. A is a variable point on x-axis and B(0,b) is a fixied point. A equilateral triangle ABC is completed with vertex C away from origin. If the locus of the point C is $\alpha x + \beta y = b$, then $\alpha^3 + \beta^3$ is

9. The portion of the line ax + by – 1 = 0, intercepted between the lines ax + y + 1 = 0 and x + by = 0 subtends a right angle at the origin and the condition in a and b is $\lambda a + b + b^2 = 0$, then find value of λ.

10. If the straight lines joining the origin and the points of intersection of the curve $5x^2 - 24xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of | k |.

11. If the points of intersection of curves $C_1 = 4y^2 - 2\lambda x^2 - 2xy - 9x + 3$ and $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ subtends a right angle at origin, then find the value of λ.

12. Let the integral points inside or on the boundary of region bounded by straight lines as shown in figure is equal to k, then $\sqrt{k}$ is equal to



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Point P(2, 3) lies on the line $4x + 3y = 17$. Then find the co-ordinates of points farthest from the line which are at 5 units distance from the P.
(A) (6, 6) (B) (6, –6) (C) (2, 0) (D) (–2, 0)

2. Find the equation of the line passing through the point (2, 3) & making intercept of length 2 units between the lines $y + 2x = 3$ & $y + 2x = 5$.
(A) $3x - 4y = 18$ (B) $x = 2$ (C) $3x + 4y = 18$ (D) $x + 2 = 0$

3. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then the co-ordinates of B and C will be
(A) $(-2, 7), (4, 3)$ (B) $(7, -2), (4, 3)$ (C) $(2, 7), (-4, 3)$ (D) $(2, -7), (3, -4)$

4. A is a point on either of two rays $y + \sqrt{3}\,|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co–ordinates of the foot of perpendicular from A on the bisector of the angle between them is/are
(A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (B) (0, 0) (C) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (D) (0, 4)

5. If one side of a square is parallel to $3x - 4y = 0$ & its area being 16 while centre being (1, 1), then find equation of sides of square.
(A) $3x - 4y + 11 = 0$ (B) $3x - 4y - 9 = 0$ (C) $4x + 3y + 3 = 0$ (D) $4x + 3y - 17 = 0$

6. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.
(A) $2x - y + 3 = 0$ (B) $x + 2y - 6 = 0$ (C) $2x + y - 7 = 0$ (D) $x - 2y - 6 = 0$

7. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then the correct statement(s) among the following is/are (O is origin)
(A) The absolute minimum value of OP + OQ, where O is origin is $18\sqrt{2}$
(B) Minimum area of ΔOPQ is 32
(C) The absolute minimum value of OP + OQ, where O is origin is 18
(D) Area of ΔOPQ is minimum for slope $\left(\frac{-1}{4}\right)$.

8. The equation of the diagonals of a rectangle are $y + 8x - 17 = 0$ and $y - 8x + 7 = 0$. If the area of the rectangle is 8 sq. units, find the equation of the sides of the rectangle.
(A) $y = 1$ (B) $y = 9$ (C) $x = 1$ (D) $x = 2$.

9. Two adjacent sides of a rhombus are $2x + 3y = a - 5$ and $3x + 2y = 4 - 2a$ and its diagonals intersect at the point (1, 2), then a can be –

(A) – 16 (B) 16 (C) $-\frac{10}{3}$ (D) $\frac{10}{3}$

10. A line $L_1 \equiv 3y - 2x - 6 = 0$ is rotated about its point of intersection with y–axis in clockwise direction to make it L_2 such that the area formed by L_1, L_2, x–axis and line $x = 5$ is $\frac{49}{3}$ sq units if its point of intersection with $x = 5$ lies below x–axis then points lying on the equation of L_2 are
(A) (3, – 1) (B) (4,2) (C) (1,1) (D) (3,3)

11. Let $D(x_4, y_4)$ be a point such that ABCD is a square & M & P are the midpoints of the sides BC & CD respectively, then
(A) Ratio of the areas of ΔAMP and the square is 3 : 8
(B) Ratio of the areas of ΔMCP & ΔAMD is 1 : 1
(C) Ratio of the areas of ΔABM & ΔADP is 1 : 1
(D) Ratio of the areas of the quandrilateral AMCP and the square is 1 : 3

12. The equations of perpendicular of the sides AB & AC of Δ ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and circumcenter is $\left(\frac{3}{2}, \frac{5}{2}\right)$, then which of the following is true.
(A) equation of median of side AB is $x - y + 1 = 0$
(B) centroid of triangle ABC is (3, 1)
(C) vertex C is (4, 0)
(D) Area of triangle ABC is 12.

13. Triangle ABC lies in the cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respecitvely and the coordinates of A are (p, q). The median to the side BC has slope – 5, then which can be corrected.
(A) $p + q = 47$ (B) $p + q = 27$ (C) $p - q = 17$ (D) $p - q = 13$

14. All the points lying on or inside the triangle formed by the points (1, 3), (5, 6) and (–1, 2) satisfy
(A) $3x + 2y \geq 0$ (B) $2x + y + 1 \geq 0$ (C) $2x + 3y - 12 \geq 0$ (D) $2x + 11 \geq 0$

15. $A \equiv (4, 2)$ and $B \equiv (2, 4)$ are two given points and a point P on the line $3x + 2y + 10 = 0$ is given then which of the following is/are true.
(A) (PA + PB) is minimum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$ (B) (PA + PB) is maximum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$
(C) |PA – PB | is maximum when P(– 22, 28) (D) (PA – PB) is minimum when P(– 22, 28).

16. A line passing through $P = (\sqrt{3}, 0)$ and making an angle of 60° with positive direction of x-axis cuts the parabola $y^2 = x + 2$ at A and B, then :
(A) $PA + PB = \frac{2}{3}$ (B) $|PA - PB| = \frac{2}{3}$
(C) $(PA)(PB) = \frac{4(2+\sqrt{3})}{3}$ (D) $\frac{1}{PA} + \frac{1}{PB} = \frac{2-\sqrt{3}}{2}$

17. Let $u \equiv ax + by + a\sqrt[3]{b} = 0$, $v \equiv bx - ay + b\sqrt[3]{b} = 0$, where $a, b \in R$ be two straight lines, then find the equations of the bisectors of the angles formed by $k_1u - k_2v = 0$ & $k_1u + k_2v = 0$ for non zero real k_1 & k_2 are :
(A) $u = 0$ (B) $k_2u + k_1v = 0$ (C) $k_2u - k_1v = 0$ (D) $v = 0$

18. The sides of a triangle are the straight line x + y = 1, 7y = x and $\sqrt{3}$ y + x = 0. Then which of the following is an interior points of triangle ?
(A) circumcentre (B) centroid (C) incentre (D) orthocentre

19. The line 'ℓ_1' passing through the point (1, 1) and the 'ℓ_2' passes through the point (– 1, 1). If the difference of the slope of lines is 2. Find the locus of the point of intersection of the ℓ_1 and ℓ_2.
(A) $x^2 = y$ (B) $y = 2 - x^2$ (C) $y^2 = x$ (D) $x = 2 - y^2$

20. The two lines pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon, then the correct statement among the following is/are
(A) Area of polygon is 6 (B) Length of its diagonals are $\sqrt{5}$ & $\sqrt{53}$
(C) Point of intersection of diagonals is (– 2, 2) (D) Polygon is parallelogram.

21. If the distance between the lines represented $9x^2 - 24xy + 16y^2 + k(6x - 8y) = 0$ is 4, then k may be
(A) 3 (B) 10 (C) –10 (D) 7

PART - IV : COMPREHENSION

Comprehenssion # 1 (Q. NO. 1 to 3)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

1. The equation of side AB is
(A) 2x + y = 4 (B) x + y – 3 = 0 (C) 4x – 2y = 0 (D) none of these

2. Co-ordinates of D are
(A) (7, –4) (B) (5, 0) (C) (7, 4) (D) (– 3, 0)

3. Height of altitude drawn from point A is (in units)
(A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C) $6\sqrt{2}$ (D) $2\sqrt{3}$

Comprehension # 2 (Q.No. 4 to 6)

If vertices of triangle are $P(p_1, p_2)$, $Q(q_1, q_2)$, $R(r_1, r_2)$, then area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ and if P, Q, R are collinear, then $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0$.

On the basis of above answer the following question.

4. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of median through A.

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) None of these

5. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of line through A and parallel to BC

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$

6. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find the equation of internal angle bisector through A

(A) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (B) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (D) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

COMPREHENSION # 3 (7-9)

The equation of pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

The equation of pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{(x-\alpha)^2 - (y-\beta)^2}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$ where (α, β) is point of intersecting of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

7. Find the combined equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$

(A) $5x^2 - 2xy - 5y^2 = 0$ (B) $5x^2 - 5y^2 + 4xy = 0$

(C) $5x^2 - 5y^2 + 2xy = 0$ (D) $5x^2 - 5y^2 - xy = 0$

8. Find the combined equation of the bisectors of the angle between the lines represented by $2x^2 - 3y^2 + xy - x + 11y - 10 = 0$

(A) $x^2 - y^2 + xy + 4x + 2 = 0$ (B) $x^2 - y^2 - 10xy + 8x + 12y - 10 = 0$

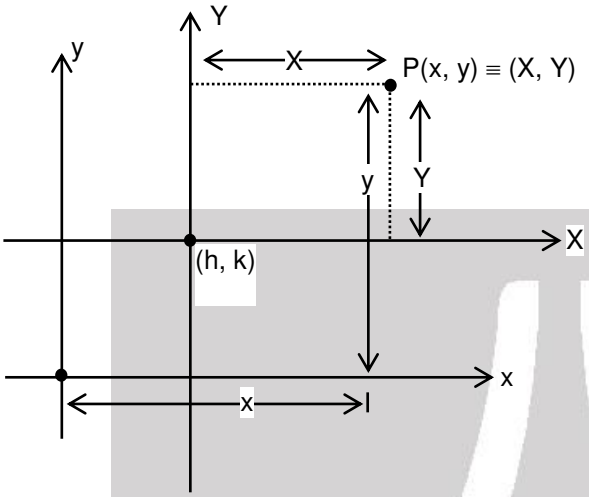
(C) $x^2 - y^2 - 5xy + 3x + 7y - 5 = 0$ (D) None of these

9. Let combined equation of lines ℓ_1 and ℓ_2 is $x^2 - 2pxy - y^2 = 0$ combined equation of lines ℓ_3 and ℓ_4 is $x^2 - 2qxy - y^2 + 2gx + 2fy + c = 0$. If angle between lines (ℓ_1, ℓ_3), (ℓ_1, ℓ_4), (ℓ_2, ℓ_3) and (ℓ_2, ℓ_4) is equal to 45°, then find the value of pq.

(A) –4 (B) 4 (C) –1 (D) 1

Comprehension (10 & 11)

Origin of coordinate system xy is shifted to (h, k) to make new coordinate system XY. X and Y are parallel to x and y. New co-ordinates of point P(x, y) are P(X, Y). x, y, X, Y are related as given below.
X = x – h
Y = y – K



10. Co-ordinates of (–7, 9) if origin is shifted to (2, 4) without changing direction of axes, are
(A) (–5,13) (B) (–7,4) (C) (–9,5) (D) (–9,13)

11. If co-ordinate axes are so translated such that ordinate of (4, 12) becomes zero while abscissa remains same. Then new coordinates of point (–8, –2) are
(A) (–8, 14) (B) (–8, 10) (C) (–8, –14) (D) (–8, –10)

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are
[IIT-JEE - 2007, P-II, (3, – 1), 81]

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

2. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-JEE - 2007, P-II, (3, – 1), 81]**

STATEMENT – 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.
because
STATEMENT – 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement - 1 is True, Statement - 2 is True; Statement-2 **is** a correct explanation for Statement - 1
(B) Statement -1 is True, Statement - 2 is True; Statement-2 is **NOT** a correct explanation for Statement - 1
(C) Statement - 1 is True, Statement - 2 is False
(D) Statement - 1 is False, Statement - 2 is True

3. Consider three points **[IIT-JEE - 2008, P-II, (3, – 1), 81]**
P = (– sin ($\beta - \alpha$), – cos β), Q = (cos($\beta - \alpha$), sin β) and R = (cos ($\beta - \alpha + \theta$), sin ($\beta - \theta$)), where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,
(A) P lies on the line segment RQ (B) Q lies on the line segment PR
(C) R lies on the line segment QP (D) P, Q, R are non-collinear

4. The locus of the orthocentre of the triangle formed by the lines **[IIT-JEE - 2009, Paper-2, (3, –1), 80]**
(1 + p) x – py + p (1 + p) = 0, (1 + q) x – qy + q(1 + q) = 0 and y = 0, where $p \neq q$, is
(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

5. A straight line **L** through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If **L** also intersects the x-axis, then the equation of **L** is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**
(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

6. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**
(A) a + b – c > 0 (B) a – b + c < 0 (C) a – b + c > 0 (D) a + b – c < 0

7. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines x – y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is **[AIEEE - 2010 (8, –2), 144]**
(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

2. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE - 2011, I(4, –1), 120]**
Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$
Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(1) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

3. The lines x + y = | a | and ax – y = 1 intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE - 2011, II(4, –1), 120]**
(1) (0, ∞) (2) [1, ∞) (3) (–1, ∞) (4) (–1, 1]

4. If A(2, –3) and B(–2, 1) are two vertices of a triangle and third vertex moves on the line 2x + 3y = 9, then the locus of the centroid of the triangle is : **[AIEEE - 2011, II(4, –1), 120]**
(1) x – y = 1 (2) 2x + 3y = 1 (3) 2x + 3y = 3 (4) 2x – 3y = 1

5. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE-2012, (4, –1)/120]**
(1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

6. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE-2012, (4, –1)/120]**

(1) $-\frac{1}{4}$ (2) – 4 (3) – 2 (4) $-\frac{1}{2}$

7. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[AIEEE - 2013, (4, –1),360]**

(1) $y = x + \sqrt{3}$ (2) $\sqrt{3}y = x - \sqrt{3}$ (3) $y = \sqrt{3}x - \sqrt{3}$ (4) $\sqrt{3}y = x - 1$

8. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[AIEEE - 2013, (4, –1),360]**

(1) $2 + \sqrt{2}$ (2) $2 - \sqrt{2}$ (3) $1 + \sqrt{2}$ (4) $1 - \sqrt{2}$

[AIEEE - 2013, (4, –1),360]

9. Let PS be the median of the triangle with vertices P(2, 2), Q (6, – 1), and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) 4x + 7y + 3 = 0 (2) 2x – 9y – 11 = 0 (3) 4x – 7y – 11 = 0 (4) 2x + 9y + 7 = 0

10. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then :

[JEE(Main) 2014, (4, – 1), 120]

(1) 3bc – 2ad = 0 (2) 3bc + 2ad = 0 (3) 2bc – 3ad = 0 (4) 2bc + 3ad = 0

11. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is **[JEE(Main) 2015, (4, – 1), 120]**

(1) 901 (2) 861 (3) 820 (4) 780

12. Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0. if its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **[JEE(Main) 2016, (4, – 1), 120]**

(1) (–3, –8) (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) (–3, –9)

13. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$ (3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

14. A straight line through a fixed point (2,3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

[JEE(Main) 2018, (4, – 1), 120]

(1) 3x + 2y = xy (2) 3x + 2y = 6xy (3) 3x + 2y = 6 (4) 2x + 3y = xy

15. Consider the set all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ? **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) The lines are not concurrent (2) The lines are all parallel

(3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$ (4) Each the line passes through the origin.

16. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is :

[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]

(1) $26x - 122y - 1675 = 0$ (2) $26x + 61y + 1675 = 0$

(3) $122y - 26x - 1675 = 0$ (4) $122y + 26x + 1675 = 0$

17. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ? **[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]**

(1) third (2) second (3) first (4) fourth

18. Let O(0,0) and A (0,1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $9x^2 - 8y^2 + 8y = 16$ (2) $8x^2 - 9y^2 + 9y = 18$

(3) $9x^2 + 8y^2 - 8y = 16$ (4) $8x^2 + 9y^2 - 9y = 18$

[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

19. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular form the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is -

(1) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$ (2) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

(3) $\sqrt{3}x+y=8$ (4) $x+\sqrt{3}y=8$

[JEE(Main) 2019, Online (12-04-19),P-2 (4, – 1), 120]

20. The locus of the mid-point of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is:

(1) $3x - 2y = 0$ (2) $3x - 3y = 0$ (3) $5x - 7y = 0$ (4) $7x - 5y = 0$

[JEE(Main) 2020, Online (07-01-20),P-2 (4, –1), 120]

Answers

EXERCISE - 1

PART - I

Section (A):

A-1. (ii) (3, 4) (6, 8) **A-2.** (i) 1:3 internally (ii) 1:2

A-3. (i) $\frac{3}{2}$ (ii) 4 : 1 internally (iii) (8, – 8) **A-4.** (7, 2) or (1, 0) **A-5.** (i) 15 sq. units (ii) |ab| sq. units

Section (B) :

B-1. (i) $y = -\frac{1}{\sqrt{3}}x - \frac{4}{\sqrt{3}}$, slope $= -\frac{1}{\sqrt{3}}$, y-intercept $= -\frac{4}{\sqrt{3}}$

(ii) $\frac{x}{-4} + \frac{y}{-4/\sqrt{3}} = 1$, x-intercept = – 4, y-intercept $= -\frac{4}{\sqrt{3}}$

(iii) $x\cos 240^\circ + y\sin 240^\circ = 2$, P = 2, $\alpha = 240^\circ$ **B-2.** 3 **B-3.** 8x + 5y + 60=0,2x+5y+30 = 0

B-4. Equations are 2x – 3y + 15 = 0, 2x + y – 5 = 0 , 6x – y – 27 = 0

Length of sides are $2\sqrt{13}$, $4\sqrt{5}$, $2\sqrt{37}$

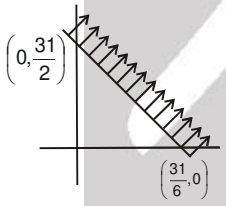
B-5. x – y + 1 = 0 **B-6.** x + y = 5, x – 7y + 3 = 0 **B-7.** 14 **B-8.** $\pi/12, 5\pi/12$

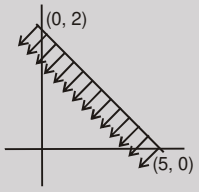
B-9. x = 3, y = 4, 9/2 sq. units **B-10.** x – 5y + 3 = 0, 5x + y – 11 = 0 **B-11.** $\left(-6, 9-2\sqrt{3}\right)$

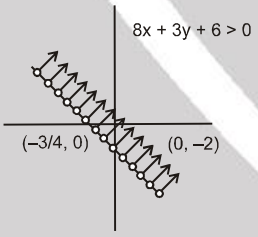
B-12. $x^2 + y^2 – 6x – 6y = 0$. **B-13.** 18 **B-14.** 7x – 4y + 25 = 0, 7x – 4y – 3 = 0, 4x + 7y = 11

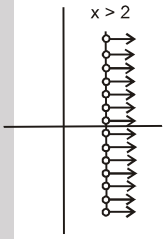
Section (C) :

C-1.

(i) 

(ii) 

(iii) 

(iv) 

C-2. Foot $\left(\frac{23}{10}, \frac{29}{10}\right)$, Image $\left(\frac{13}{5}, \frac{14}{5}\right)$, x + 3y – 11 = 0 **C-3.** $\frac{1+\sqrt{10}}{3}$ **C-4.** $\left(\frac{3}{25}, \frac{4}{25}\right)$

C-5. Inside **C-6.** 3 sq. unit **C-7.** 2 **C-8.** x + 7y + 12 = 0, x + 7y – 8 = 0

Section (D) :

D-1. (i) (7, 8) (ii) $\left(\frac{21}{2}, \frac{8}{3}\right)$ (iii) (7, 9) (iv) (27, –21) **D-2.** 7

D-3. 2x + 3y + 8 = 0, 4x – 3y – 7 = 0, 8x + 3y + 9 = 0 , orthocentre $\left(-\frac{1}{6}, -\frac{23}{9}\right)$ **D-4.** (0, 0)

D-6. $\frac{3}{x} + \frac{2}{y} = 3$ **D-7.** $(3x – 1)^2 + 9y^2 = a^2 + b^2$ **D-9.** $x^2 + y^2 = a^2 + b^2$ **D-10.** y = 2x+1 or y=–2x+ 1

Section (E) :

E-1. acute 2x + y – 3 = 0, obtuse x – 2y + 1 = 0, origin lies in obtuse angle bisector.

E-2. 9 x – 7 y = 1, 7 x + 9 y = 73 **E-3.** 5x + 5y – 3 = 0 **E-4.** $\frac{6}{5}, \frac{1}{2}, -3$

E-5. 0 **E-6.** (i) $43x - 29y = 71$ (ii) $x + y + 2 = 0$

E-7. $2xy(ma - b\ell) + x(an - \ell c) + y(mc - bn) = 0$.

Section (F) :

F-2. $\lambda = 2$, (i) $3x - y + 2 = 0$, $4x - 2y + 1 = 0$, (ii) point of intersection $\left(-\frac{3}{2}, -\frac{5}{2}\right)$, (iii) $\tan^{-1}\left(\frac{1}{7}\right)$,

F-3. (i) h = 1 (ii) k = 1 **F-4.** $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

F-5. $a^2 + b^2 = (a')^2 + (b')^2 \Rightarrow (a + a')x + (b + b')y + c + c' = 0 \Rightarrow (a - a')x + (b - b')y = 0$

PART – II

Section (A):

A-1. (C) **A-2.** (B) **A-3.** (B) **A-4.** (A) **A-5.** (A)

Section (B) :

B-1. (D) **B-2.** (B) **B-3.** (C) **B-4.** (B) **B-5.** (D) **B-6.** (B) **B-7.** (B)
B-8. (B) **B-9.** (A) **B-10.** (B) **B-11.** (B) **B-12.** (C) **B-13.** (B)

Section (C) :

C-1. (B) **C-2.** (B) **C-3.** (A) **C-4.** (C) **C-5.** (A) **C-6.** (C)

Section (D) :

D-1. (A) **D-2.** (D) **D-3.** (C) **D-4.** (B) **D-5.** (B) **D-6.** (A) **D-7.** (A)
D-8. (A)

Section (E) :

E-1. (A) **E-2.** (C) **E-3.** (A) **E-4.** (D) **E-5.** (D) **E-6.** (B) **E-7.** (D)
E-8. (B) **E-9.** (C)

Section (F) :

F-1. (D) **F-2.** (C) **F-3.** (A)

PART – III

1. (A) $\rightarrow$ (q, s) (B) $\rightarrow$ (r, s) (C) $\rightarrow$ (r, s) (D) $\rightarrow$ (r, s) **2.** (A) $\rightarrow$ (p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

EXERCISE - 2

PART – I

1. (C) **2.** (C) **3.** (D) **4.** (A) **5.** (D) **6.** (C) **7.** (B)
8. (A) **9.** (C) **10.** (A) **11.** (A) **12.** (C) **13.** (C) **14.** (A)
15. (B) **16.** (A) **17.** (B) **18.** (A) **19.** (B) **20.** (A) **21.** (A)

PART – II

1. 00.00 **2.** 02.98 **3.** 18.00 **4.** 01.00 **5.** **33.80** **6.** 04.72 or 04.73
7. 04.00 **8.** 04.19 or 04.20 **9.** 02.00 **10.** 02.60 **11.** 09.50 **12.** 18.19

PART – III

1. (AD) **2.** (BC) **3.** (B) **4.** (B) **5.** (ABCD) **6.** (ACD) **7.** (BCD)
8. (ABCD) **9.** (AD) **10.** (AC) **11.** (AC) **12.** (C) **13.** (ABD) **14.** (ABD)
15. (AC) **16.** (BC) **17.** (AD) **18.** (BC) **19.** (AB) **20.** (ABD) **21.** (BC)

PART – IV

1. (A) **2.** (B) **3.** (C) **4.** (B) **5.** (A) **6.** (C) **4.** (B)
5. (A) **6.** (C) **7.** (A) **8.** (D) **9.** (C) **10.** (C) **11.** (C)

EXERCISE - 3

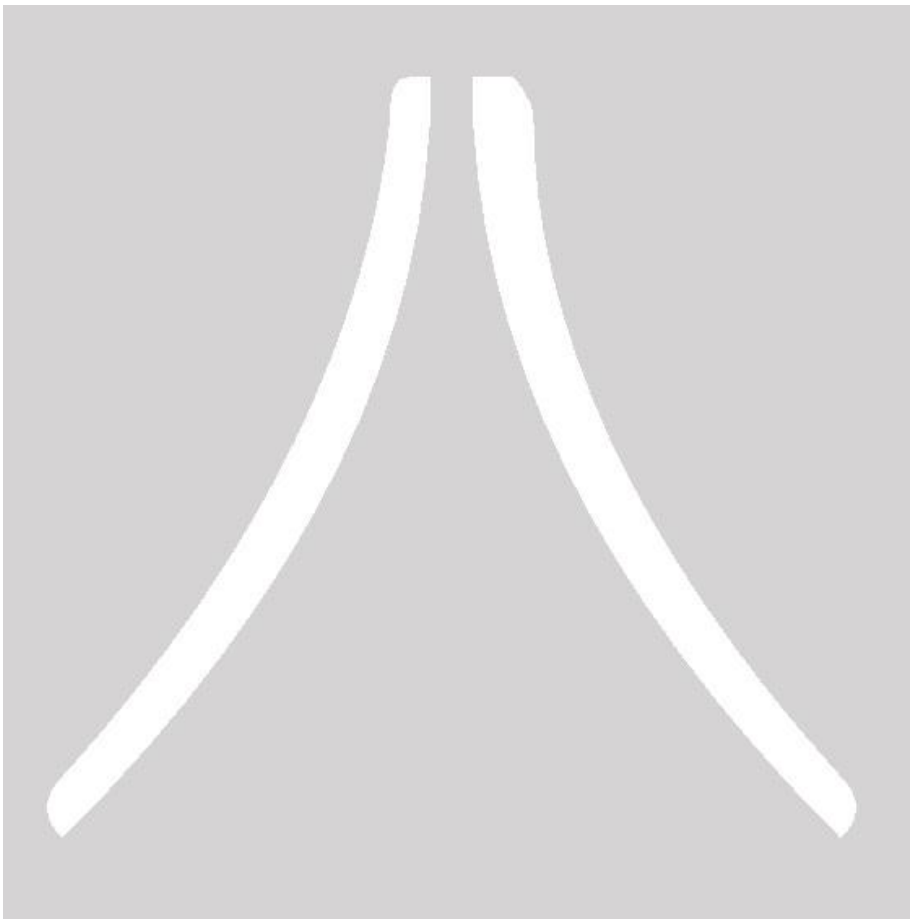
PART – I

1. (C) **2.** (C) **3.** (D) **4.** (D) **5.** (B) **6.** (A) or (C) or Bonus
7. (6)

PART – II

1. (3) **2.** (3) **3.** (2) **4.** (2) **5.** (3) **6.** (3) **7.** (2)
8. (2) **9.** (4) **10.** (1) **11.** (4) **12.** (2) **13.** (4) **14.** (1)

15. (3) **16.** (1) **17.** (2) **18.** (3) **19.** (2) **20.** (3)



High Level Problems (HLP)

Marked Questions may have for Revision Questions.

1. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.

2. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a+b)=ab(x+y)$.

3. From the vertices A, B, C of a triangle ABC, perpendiculars AD, BE, CF are drawn to any straight line. Show that the perpendiculars from D, E, F to BC, CA, AB respectively are concurrent.

4. A triangle is formed by the lines whose equations are AB : x + y – 5 = 0, BC : x + 7y – 7 = 0 and CA : 7x + y + 14 = 0. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

5. The coordinates of the feet of $\perp$ from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). Then find the coordinates of a vertices of the Δ

6. Let P is any point inside the triangle ABC of side lengths 6, 5, 5 units and p_1, p_2, p_3 be the lengths of perpendiculars drawn from P to the sides of triangle. Find the maximum value of $p_1.p_2.p_3$.

7. Let in ΔPAB, A is (0, 0), B is (a, 0) and P is variable such that $\angle$PBA is equal to three times $\angle$PAB the, find the locus of P.

8. Through a fixed point any straight line is drawn meeting two given parallel straight lines in P and Q, through P and Q straight lines are drawn in fixed directions, meeting in R. Prove that the locus of R is straight line.

9. Through the origin O a straight line is drawn to cut the lines $y = m_1x + C_1$ and $y = m_2x + C_2$ at Q and R. respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean of OQ and OR.

10. The sides of a triangle are $L_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0$ for r = 1, 2, 3.
Show that its orthocentre is given by
$L_1\cos(\alpha_2-\alpha_3) = L_2\cos(\alpha_3-\alpha_1) = L_3\cos(\alpha_1-\alpha_2)$.

11. A line passes through a fixed point R intersecting a fixed line at P. A point Q on RP such that $\frac{RP}{RQ}$ is constant. Then show that locus of Q is a straight line.

12. A triangle ABC with a = 8, b = 6 and c = 10 slides on the coordinate axes with vertices A and B on the x–axis and the y–axis respectively. Find the locus of the vertex C.

13. The line $L_1 \equiv 4x + 3y - 12 = 0$ intersects the x and the y-axis at A and B respectively. A variable line perpendicular to L_1 intersects the x and the y-axis at P and Q respectively. Find the locus of the circumcentre of triangle ABQ.

14. Show that the orthocentre of Δ formed by the straight lines, $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is a point (x', y') such that $\frac{x'}{\ell}=\frac{y'}{m}=\frac{a+b}{am^2-2h\ell m+b\ell^2}$.

15. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co–ordinates is δ, find the combined equation of these straight lines.

16. A point moves so that the distance between the feet of the perpendiculars from it on the lines $ax^2 + 2h xy + by^2 = 0$ is a constant 2d. Show that the equation to its locus is,
$(x^2 + y^2)(h^2 - ab) = d^2\{(a - b)^2 + 4h^2\}$.

17. Show that the pair of lines given by $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is equally inclined to the pair given by $ax^2 + 2hxy + by^2 = 0$.

18. All the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence.

19. The straight lines $(A^2 - 3B^2)x^2 + 8AB\,xy + (B^2 - 3A^2)y^2 = 0$ form a Δ with the line $Ax + By + C = 0$, then prove that

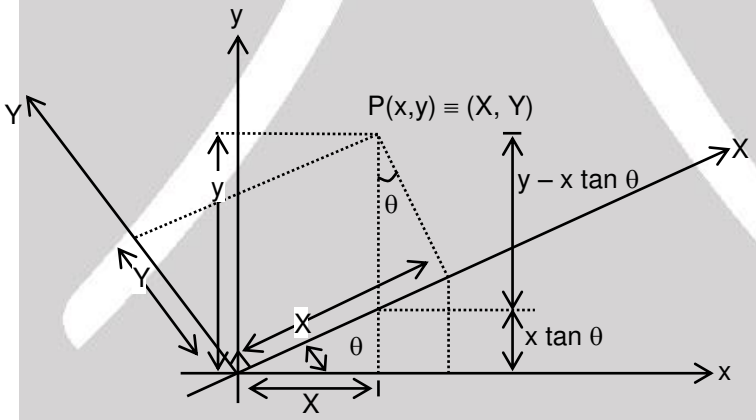
(i) Area of Δ is $\dfrac{C^2}{\sqrt{3}\left(A^2 + B^2\right)}$

(ii) Δ is equilateral

(iii) The orthocentre of Δ does not lie on one of its vertexs

Comprehension (Q. 20 & 21)

If coordinate system xy is being rotated through an angle θ in anti clock wise direction about the origin as shown in the diagram, Coordinates of P(x, y) has been change to P(X, Y) in new coordinate system XY, then x, y, X, Y are related as given below.



$X = x\sec\theta + (y - x\tan\theta)\sin\theta$ and $Y = (y - x\tan\theta)\cos\theta$

$= x\sec\theta + y\sin\theta - \dfrac{x\sin^2\theta}{\cos\theta}$ $= y\cos\theta - x\sin\theta$

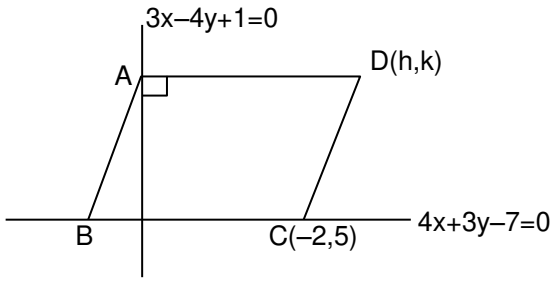
$\boxed{X = x\cos\theta + y\sin\theta}$ $\boxed{Y = -x\sin\theta + y\cos\theta}$

20. If the axes are rotated through 60º in anticlockwise direction about origin. Find co-ordinats of point (2, 6) in new co-ordinate axes.

21. If axes are rotated through an acute angle in clockwise direction about origin so that equation $x^2 + 2xy + y^2 - 2x + 2y = 0$ becomes free from xy in its new position, then find equation in new position

22. Find the acute angle between two straight lines passing through the point M(−6, −8) and the points in which the line segment $2x + y + 10 = 0$ enclosed between the co–ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x–axis to the point of intersection with the y–axis.

23_. Let A lies on $3x - 4y + 1 = 0$. B lies on $4x + 3y - 7 = 0$ and C is $(-2, 5)$. If ABCD is rhombus, then find locus of D.



DA = DC

$(h + 2)^2 + (A - 5)^2 = \left(\frac{3h - 4k + 1}{5}\right)^2$

$25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$

24_. Let D is point on line $\ell_1 : x + y - 2 = 0$ and S(3, 3) is fixed point. ℓ_2 is the line perpendicular to DS and passing through S. If M is another point on line ℓ_1 (other than D), then find locus of point of intersection of ℓ_2 and angle bisector of $\angle$MDS.

25_. A variable line cuts the line $2y = x - 2$ and $2y = -x + 2$ in points A and B respectively. If A lies in first quadrant, B lies in 4th quadrant and area of ΔAOB is 4, then find locus of

(i) mid point of AB (ii) centroid of ΔOAB

26. An equilateral triangle PQR is formed where P (1, 3) is fixed point and Q is moving point on line $x = 3$. Find the locus of R.

Answers

1. $x + y + \frac{1}{\sqrt{2}} = 0.$ **4.** $3x + 6y - 16 = 0; 8x + 8y + 7 = 0 ; 12x + 6y - 11 = 0$

5. (5, 10). (50, – 5), (15, 30) **6.** $p_1p_2p_3 \le \frac{256}{75}$ **7.** $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0.$

9. $(y - m_1x)\ (y - m_2x) = c_1c_2$ **12.** $\frac{y}{x} = \frac{3}{4}$ or $3x - 4y = 0$ **13.** $6x - 8y + 7 = 0$

15. $(y_1^2 - \delta^2)\ x^2 - 2\ x_1y_1\ xy + (x_1^2 - \delta^2)\ y^2 = 0$ **18.** (1, – 2), yes (1/3, – 2/3)

20. $(1 + 3\sqrt{3}, -\sqrt{3} + 3)$ **21.** $x^2 + \sqrt{2}y = 0$ **22.** $\pi/4$

23. $25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$ **24.** $(x - 3)^2 + (y - 3)^2 = \left(\frac{x + y - 2}{\sqrt{2}}\right)^2$

25. (i) $(x - 1)^2 - 4y^2 = 9$ (ii) $(x - 2/3)^2 + 4y^2 = 4$ **26.** $(x - 2) = \pm\sqrt{3}\ (y - 3 \pm \sqrt{3})$

Exercise-1

Marked questions are recommended for Revision.
चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

PART - I : SUBJECTIVE QUESTIONS

भाग - I : विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

खण्ड **(A):** दुरी सूत्र, विभाजन सूत्र, त्रिभुज का क्षेत्रफल, बहुपद, संरेखीयता, प्रवणता

A-1.(i) Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral triangle whose side is 2a.

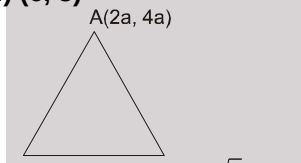
सिद्ध कीजिए कि बिन्दु (2a, 4a), (2a, 6a) एवं $(2a + \sqrt{3}a, 5a)$ एक समबाहु त्रिभुज के शीर्ष है जिसकी भुजा 2a है।

(ii) Find the points which trisect the line segment joining the points (0, 0) and (9, 12).

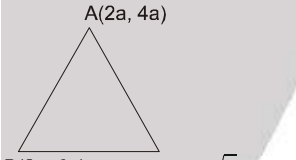
बिन्दुओं (0, 0) एवं (9, 12) को मिलाने वाले रेखाखण्ड को समत्रिभाजित करने वाले बिन्दुओं के निर्देशांक हैं –

Ans. **(ii) (3, 4) (6, 8)**

Sol. **(i)**



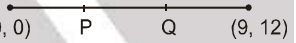
AB= BC = CA = 2a hence triangle is equilateral.



AB= BC = CA = 2a अतः त्रिभुज समबाहु त्रिभुज होगा।

(ii) Let required point is P & Q

P divides in 1 : 2 $\quad P\left(\frac{9+2\times0}{1+2}, \frac{1\times12+2\times0}{1+2}\right) \equiv (3, 4)$



Q divides in 2 : 1 $\quad$ Hence $Q\left(\frac{2\times9+1\times0}{2+1}, \frac{2\times12+1\times0}{2+1}\right) \equiv Q\ (6, 8)$

माना अभीष्ट बिन्दु P एवं Q है।

P, 1 : 2 के अनुपात में विभाजित करेगा। $P\left(\frac{9+2\times0}{1+2}, \frac{1\times12+2\times0}{1+2}\right) \equiv (3, 4)$

Q, 2 : 1 के अनुपात में विभाजित करेगा। अतः $Q\left(\frac{2\times9+1\times0}{2+1}, \frac{2\times12+1\times0}{2+1}\right) \equiv Q\ (6, 8)$

A-2. **(i)** In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (– 7, 9)?

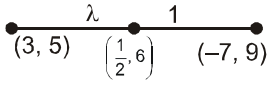
बिन्दुओं (3,5) एवं (–7,9) को मिलाने वाले रेखाखण्ड को बिन्दु $\left(\frac{1}{2}, 6\right)$ किस अनुपात में विभाजित करता हैं?

(ii) In which ratio P(2a – 2, 4a – 6)) divides Q (2a – 3, 3a – 7) and R(2a, 6a – 4).

बिन्दुओं Q (2a – 3, 3a – 7) और R(2a, 6a – 4) को मिलाने वाले रेखाखण्ड को बिन्दु P(2a – 2, 4a – 6) किस अनुपात में विभाजित करता हैं ?

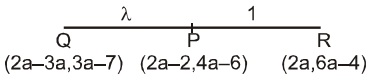
Ans. **(i)** 1:3 internally आन्तरिक **(ii)** 1:2

Sol. **(i)** Let $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (–7, 9) in $\lambda : 1$



$$\frac{1}{2} = \frac{-7\lambda + 3}{\lambda + 1} \quad \Rightarrow \quad \lambda = 1 : 3$$

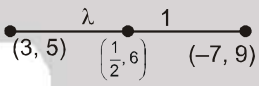
(ii)



$$\frac{2a\lambda + 2a - 3}{\lambda + 1} = 2a - 2 \qquad \frac{6a\lambda - 4\lambda + 3a - 7}{\lambda + 1} = 4a - 6$$

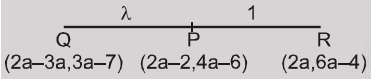
$$\lambda = \frac{1}{2}. \qquad \lambda = \frac{1}{2}.$$

Hindi. **(i)** माना $\left(\frac{1}{2}, 6\right)$ बिन्दुओं (3, 5) एवं (–7, 9) को मिलाने से प्राप्त रेखाखण्ड को $\lambda : 1$ के अनुपात में विभाजित करता है।



$$\frac{1}{2} = \frac{-7\lambda + 3}{\lambda + 1} \quad \Rightarrow \quad \lambda = 1 : 3$$

(ii)



$$\frac{2a\lambda + 2a - 3}{\lambda + 1} = 2a - 2 \qquad \frac{6a\lambda - 4\lambda + 3a - 7}{\lambda + 1} = 4a - 6$$

$$\lambda = \frac{1}{2}. \qquad \lambda = \frac{1}{2}.$$

A-3. (i) Find the value of λ such that points P(1, 2), Q(– 2, 3) and R(λ + 1, λ) are not forming a triangle ?

λ का मान ज्ञात कीजिए जब कि बिन्दु P(1, 2), Q(– 2, 3) तथा R(λ + 1, λ) एक त्रिभुज नहीं बनाती है ?

(ii) Find the ratio in which the line segment joining of the points (1, 2) and (– 2, 3) is divided by the line 3x + 4y = 7

(1, 2) तथा (– 2, 3) को मिलाने वाले रेखा खण्ड को सरल रेखा 3x + 4y = 7 जिस अनुपात में विभाजित करती है, वह अनुपात ज्ञात कीजिये।

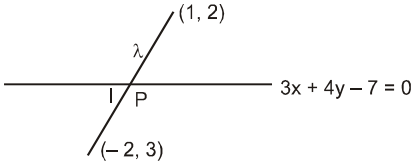
(iii) Find the harmonic conjugate of the point R (5, 1) with respect to points P (2, 10) and Q (6, – 2).

बिंदुओं P (2, 10) तथा Q (6, –2) के सापेक्ष बिंदु R(5, 1) का हरात्मक संयुग्मी ज्ञात कीजिये।

Ans. **(i)** $\frac{3}{2}$ **(ii)** 4 : 1 internally (अंतः विभाजन) **(iii)** (8, – 8).

Sol. **(i)** $\begin{vmatrix} 1 & 2 & 1 \\ -2 & 3 & 1 \\ \lambda+1 & \lambda & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad -4\lambda + 6 = 0 \quad \Rightarrow \quad \lambda = \frac{3}{2}.$

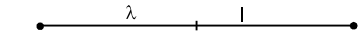
(ii) Let required point is P. coordinate of P $\left(\frac{-2\lambda+1}{\lambda+1}, \frac{3\lambda+2}{\lambda+1}\right)$



P lies on given line $3\left(\frac{-2\lambda+1}{\lambda+1}\right) + 4\left(\frac{3\lambda+2}{\lambda+1}\right) - 7 = 0$

$\Rightarrow$ $-6\lambda + 3 + 12\lambda + 8 - 7\lambda - 7 = 0 \Rightarrow -\lambda + 4 = 0 \Rightarrow \lambda = 4$

(iii) Let R(5, 1) divides line segment joining P(2,10) and Q(6, – 2) in $\lambda : 1$



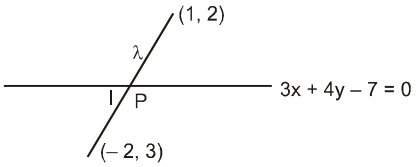
$5 = \frac{6\lambda+2}{\lambda+1} \Rightarrow \lambda = 3$

Hence Harmonic congruent divides in 3 : 1 externaly

Hence required part is $\left(\frac{18-2}{3-1}, \frac{-6-10}{3-1}\right) = (8, -8)$

Hindi **(i)** **(i)** $\begin{vmatrix} 1 & 2 & 1 \\ -2 & 3 & 1 \\ \lambda+1 & \lambda & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad -4\lambda + 6 = 0 \quad \Rightarrow \quad \lambda = \frac{3}{2}.$

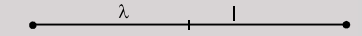
(ii) माना अभीष्ट बिन्दु P है। P के निर्देशांक $\left(\frac{-2\lambda+1}{\lambda+1}, \frac{3\lambda+2}{\lambda+1}\right)$



P, दी गई रेखा पर स्थित है $3\left(\frac{-2\lambda+1}{\lambda+1}\right) + 4\left(\frac{3\lambda+2}{\lambda+1}\right) - 7 = 0$

$\Rightarrow -6\lambda + 3 + 12\lambda + 8 - 7\lambda - 7 = 0 \Rightarrow -\lambda + 4 = 0 \Rightarrow \lambda = 4$

(iii) माना R(5, 1) बिन्दुओं P(2,10) और Q(6, – 2) को मिलाने वाले रेखाखण्ड को $\lambda : 1$ में विभाजित करता है।



$5 = \frac{6\lambda+2}{\lambda+1} \Rightarrow \lambda = 3$

अतः हरात्मक संयुग्मी, बाहय रूप से 3 : 1 में विभाजित करता है।

अतः अभीष्ट बिन्दु $\left(\frac{18-2}{3-1}, \frac{-6-10}{3-1}\right) = (8, -8)$ है।

A-4. A and B are the points (3, 4) and (5, – 2) respectively. Find the co-ordinates of a point P such that PA = PB and the area of the triangle PAB = 10.

बिन्दु A एवं B क्रमशः (3, 4) एवं (5, – 2) है। बिन्दु P के निर्देशांक ज्ञात कीजिए जबकि PA = PB एवं त्रिभुज PAB का क्षेत्रफल 10 वर्ग इकाई है।

Ans. (7, 2) or या (1, 0)

Sol. Let coordinates of P(x,y)

given PA = PB $\Rightarrow$ $(x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2 \Rightarrow 4x - 12y = 4$

$\Rightarrow x - 3y = 1$ ...(i)

$\left|\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix}\right| = 10 \Rightarrow 6x + 2y - 26 = \pm 20 \Rightarrow 3x + y - 13 = \pm 10$

$\Rightarrow 3x + y = 23$ ...(ii)

$\Rightarrow 3x + y = 3$ ...(iii)

Solving (i) and (ii) we get (7, 2) ; Solving (i) and (iii) we get (1, 0)

Hindi. माना P के निर्देशांक (x,y) है।

दिया गया है PA = PB $\Rightarrow$ $(x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2 \Rightarrow 4x - 12y = 4$

$\Rightarrow x - 3y = 1$ ...(i)

$\left|\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix}\right| = 10 \Rightarrow 6x + 2y - 26 = \pm 20 \Rightarrow 3x + y - 13 = \pm 10$

$\Rightarrow 3x + y = 23$ ...(ii) $\Rightarrow 3x + y = 3$ ...(iii)

(i) और (ii) को हल करने पर (7, 2) प्राप्त होता है।

(i) और (iii) को हल करने पर (1, 0) प्राप्त होता है।

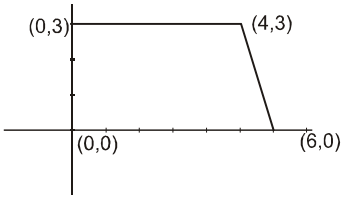
A-5. Find the area of the quadrilateral with vertices as the points given in each of the following :

निम्नलिखित शीर्षों वाले चतुर्भुजों का क्षेत्रफल ज्ञात कीजिए :

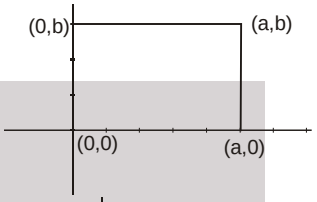
(i) (0, 0), (4, 3), (6, 0), (0, 3) (ii) (0, 0), (a, 0), (a, b), (0, b)

Ans. (i) 15 वर्ग इकाई (ii) |ab| वर्ग इकाई

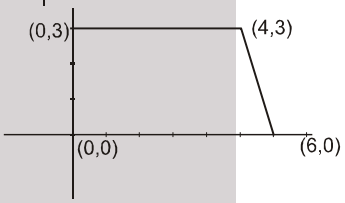
Sol. (i) Area = $\left|\frac{1}{2}\begin{vmatrix}0 & 0 & 4 & 6 & 0\\ 0 & 3 & 3 & 0 & 0\end{vmatrix}\right|$ = 15 sq. units



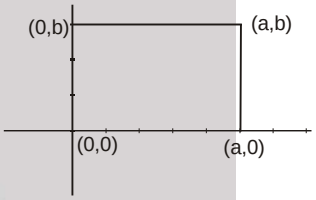
(ii) Given figure is rectangle. Hence area = |ab| sq. units



Sol. (i) क्षेत्रफल = $\left|\frac{1}{2}\begin{vmatrix}0 & 0 & 4 & 6 & 0\\ 0 & 3 & 3 & 0 & 0\end{vmatrix}\right|$ = 15 वर्ग इकाई



(ii) दिया गया चित्र, आयत है। अतः क्षेत्रफल = |ab| वर्ग इकाई



Section (B) : Different forms of straight lines and Angle between lines
खण्ड (B) : त्रिभुज और बहुभुज का क्षेत्रफल, सरल रेखाओं के विभिन्न रूप और रेखाओं के मध्य कोण

B-1. Reduce $x + \sqrt{3}\ y + 4 = 0$ to the :

(i) Slope intercepts form and find its slope and y-intercept.

(ii) Intercepts form and find its intercepts on the axes.

(iii) Normal form and find values of P and α .

रेखा $x + \sqrt{3}\, y + 4 = 0$ को

(i) प्रवणता–अंतःखण्ड रूप में परिवर्तित करो तथा इसकी प्रवणता व y-अंतःखण्ड ज्ञात कीजिए।

(ii) अंतःखण्ड रूप में परिवर्तित करो तथा इसके अक्षों पर अंतःखण्ड भी ज्ञात कीजिए।

(iii) अभिलम्ब रूप में परिवर्तित करो तथा इसके लिये P व α के मान ज्ञात कीजिए।

Ans. (i) $y = -\frac{1}{\sqrt{3}}\ x - \frac{4}{\sqrt{3}}$, slope ढाल $= -\frac{1}{\sqrt{3}}$, y-intercept अंतःखण्ड $= -\frac{4}{\sqrt{3}}$

(ii) $\frac{x}{-4} + \frac{y}{-4/\sqrt{3}} = 1$, x-intercept अंतःखण्ड $= -4$, y-intercept अंतःखण्ड $= -\frac{4}{\sqrt{3}}$

(iii) $x \cos 240^\circ + y \sin 240^\circ = 2$, $P = 2$, $\alpha = 240^\circ$

Sol. Obvious स्पष्टतः

B-2. Find number of straight line passing through (2, 4) and forming a triangle of 16 sq. cm with the coordinate axis.

सरल रेखाओं की संख्या ज्ञात कीजिए जो (2, 4) से गुजरती है तथा निर्देशांक अक्षों के साथ 16 वर्ग सेमी का त्रिभुज बनाती है।

Ans. 3

Sol. Obvious स्पष्टतया

B-3. Find the equation of the straight line that passes through the point A(– 5, – 4) and is such that the portion intercepted between the axes is divided by the point A in the ratio 1 : 2 (internally).
बिन्दु A (– 5, – 4) से गुजरने वाली तथा इस बिन्दु A पर अक्षों के मध्य 1 : 2 (आन्तरिक) में विभाजित करने वाली सरल रेखा का समीकरण ज्ञात कीजिए।

Ans. $8x + 5y + 60 = 0,\ 2x + 5y + 30 = 0$

Sol. Let equation of line $\frac{x}{a}+\frac{y}{b} = 1$ and A (a, 0), B (0, b) are intersecting points on axes P(– 5, – 4) divides AB in the ratio 1 : 2.

मानाकि रेखा का समीकरण $\frac{x}{a}+\frac{y}{b} = 1$ और A (a, 0), B (0, b) अक्षों पर दो प्रतिच्छेदी बिन्दु है जिसमें P(– 5, – 4), AB को 1 : 2 में विभाजित करता है।

$\therefore \quad P\left(\frac{2a}{3}, \frac{b}{3}\right) \Rightarrow \frac{2a}{3} = -5$ and तथा $\frac{b}{3} = -4 \Rightarrow a = \frac{-15}{2}$, b = 12

$\therefore \quad \frac{x}{\left(\frac{-15}{2}\right)} + \frac{y}{(-12)} = 1 \quad \Rightarrow \quad 8x + 5y - 60 = 0.$

For 2 : 1 $\frac{a}{3} = -5$ and $\frac{2b}{3} = -4$; a = – 15, b = – 6 $\Rightarrow \quad \frac{x}{-15} + \frac{y}{-6} = 1$

$\Rightarrow \quad 6x + 15y + 90 = 0 \quad \Rightarrow \quad 2x + 5y + 30 = 0$

B-4. The co-ordinates of the mid-points of the sides of a triangle are (2, 1), (5, 3) and (3, 7). Find the length and equation of its sides.
त्रिभुज की भुजाओं के मध्य बिन्दुओं के निर्देशांक (2, 1), (5, 3) एवं (3, 7) हैं। इसकी भुजाओं के समीकरण तथा लम्बाईयाँ ज्ञात कीजिए।

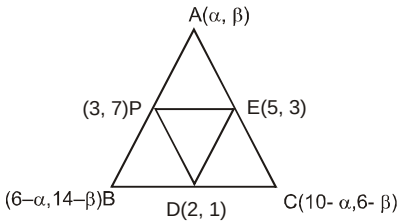
Ans. Equations are $2x - 3y + 15 = 0,\ 2x + y - 5 = 0,\ 6x - y - 27 = 0$

Length of sides are $2\sqrt{13},\ 4\sqrt{5},\ 2\sqrt{37}$

Sol. Let coordinate of A is (α, β)

$\frac{6-\alpha+10-\alpha}{2} = 2 \Rightarrow 16 - 2\alpha = 4 \Rightarrow \alpha = 6$

$\frac{14-\beta+6-\beta}{2} = 1 \Rightarrow 2\beta = 18 \Rightarrow \beta = 9$



Hence A(6, 9), B(0, 5), c(4, – 3)

Length of sides AB = $\sqrt{36+16}$ = $2\sqrt{13}$; BC = $\sqrt{16+64}$ = $4\sqrt{5}$

AC = $\sqrt{4+144}$ = $\sqrt{148}$ = $2\sqrt{37}$

and their equations are AB : 2x – 3y + 15 = 0

BC : 2x + y – 5 = 0

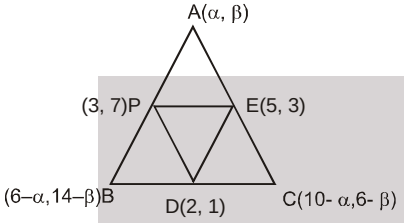
AC : 6x – y – 27 = 0

Sol. माना बिन्दु A के निर्देशांक (α, β) है।

$\frac{6-\alpha+10-\alpha}{2} = 2 \Rightarrow 16 - 2\alpha = 4 \Rightarrow \alpha = 6$

$\frac{14-\beta+6-\beta}{2} = 1 \Rightarrow 2\beta = 18 \Rightarrow \beta = 9$

अतः A(6, 9), B(0, 5), c(4, – 3)



भुजाओं की लम्बाईयाँ AB = $\sqrt{36+16}$ = $2\sqrt{13}$; BC = $\sqrt{16+64}$ = $4\sqrt{5}$

AC = $\sqrt{4+144}$ = $\sqrt{148}$ = $2\sqrt{37}$

इन भुजाओं की समीकरण AB : 2x – 3y + 15 = 0

BC : 2x + y – 5 = 0

AC : 6x – y – 27 = 0 होंगे।

B-5. Find the straight line cutting an intercept of one unit on negative x-axis and inclined at 45° (in anticlockwise direction) with positive direction of x-axis

उस सरल रेखा का समीकरण ज्ञात कीजिए जो ऋणात्मक x-अक्ष पर एक इकाई का अन्तखण्ड काटती है तथा x-अक्ष की धनात्मक दिशा से 45° का वामावर्त दिशा में कोण बनाती है

Ans. x – y + 1 = 0

Sol. Let line is y = mx + c, then m = tan 45° = 1 and c = 1

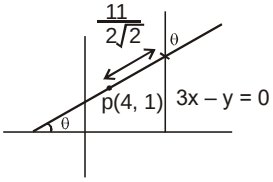
माना रेखा y = mx + c है, तब m = tan 45° = 1 तथा c = 1

B-6. Through the point P(4, 1) a line is drawn to meet the line 3x – y = 0 at Q where PQ = $\frac{11}{2\sqrt{2}}$. Determine the equation of line.

बिन्दु P(4, 1) से एक रेखा इस प्रकार खींची जाती है कि यह रेखा 3x – y = 0 को बिन्दु Q पर मिलती है जहाँ PQ = $\frac{11}{2\sqrt{2}}$. इस रेखा का समीकरण ज्ञात करो।

Ans. x + y = 5, x – 7y + 3 = 0

Sol. By parametric form Q $\left(4+\frac{11}{2\sqrt{2}}\cos\theta, 1+\frac{11}{2\sqrt{2}}\sin\theta\right)$



it lies on 3x – y = 0 $\Rightarrow$ $12 + \frac{33}{2\sqrt{2}}\cos\theta - 1 - \frac{11}{2\sqrt{2}}\sin\theta = 0$

$\Rightarrow \quad 1 + \frac{3}{2\sqrt{2}}\cos\theta - \frac{\sin\theta}{2\sqrt{2}} = 0 \quad \Rightarrow \quad 3\cos\theta - \sin\theta = -2\sqrt{2}$

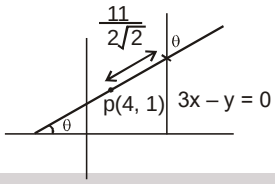
squaring both sides

$9\cos^2\theta + \sin^2\theta - 6\sin\theta\cos\theta = 8(\sin^2\theta + \cos^2\theta) \quad \Rightarrow \quad \cos^2\theta - 6\sin\theta\cos\theta - 7\sin^2\theta = 0$

$7\tan^2\theta + 6\tan\theta - 1 = 0 \quad \Rightarrow \quad \tan\theta = -1, \frac{1}{7}.$

Hence required line are $x + y = 5, x - 7y + 3 = 0$

Hindi. प्राचलिक रूप में $Q\left(4+\frac{11}{2\sqrt{2}}\cos\theta, 1+\frac{11}{2\sqrt{2}}\sin\theta\right)$



यह रेखा $3x - y = 0$ पर स्थित है $\Rightarrow \quad 12 + \frac{33}{2\sqrt{2}}\cos\theta - 1 - \frac{11}{2\sqrt{2}}\sin\theta = 0$

$\Rightarrow \quad 1 + \frac{3}{2\sqrt{2}}\cos\theta - \frac{\sin\theta}{2\sqrt{2}} = 0 \quad \Rightarrow \quad 3\cos\theta - \sin\theta = -2\sqrt{2}$

वर्ग करने पर

$9\cos^2\theta + \sin^2\theta - 6\sin\theta\cos\theta = 8(\sin^2\theta + \cos^2\theta) \quad \Rightarrow \quad \cos^2\theta - 6\sin\theta\cos\theta - 7\sin^2\theta = 0$

$7\tan^2\theta + 6\tan\theta - 1 = 0 \quad \Rightarrow \quad \tan\theta = -1, \frac{1}{7}.$

अतः अभीष्ट रेखा $x + y = 5, x - 7y + 3 = 0$

B-7. A line having slope '1' is drawn from a point A(–3, 0) cuts a curve $y = x^2 + x + 1$ at P & Q. Find $\ell(AP)\ \ell(AQ)$.

एक सरल रेखा जिसकी प्रवणता '1' है, बिन्दु A(–3, 0) से खीचीं जाती है जो वक्र $y = x^2 + x + 1$ को P तथा Q पर काटती है। $\ell(AP)\ \ell(AQ)$. का मान ज्ञात कीजिए।

Ans. 14

Sol. $\left(-3+\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$

$$\frac{r}{\sqrt{2}} = \left(-3+\frac{r}{\sqrt{2}}\right)^2 + \left(-3+\frac{r}{\sqrt{2}}\right) + 1$$

$$\left(\frac{r}{\sqrt{2}} - 3\right)^2 = 2$$

$$\frac{r^2}{2} + 9 - 6\frac{r}{\sqrt{2}} = 2 \quad \Rightarrow \quad \frac{r^2}{2} - \frac{6r}{\sqrt{2}} + 7 = 0$$

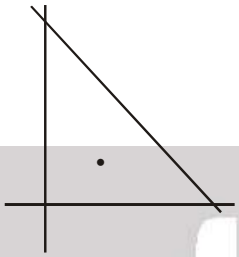
$r_1 . r_2 = \frac{7}{\frac{1}{2}} = 14$

B-8. Find the direction in which a straight line must be drawn through the point (1, 2), so that its point of intersection with the line $x + y = 4$ may be at a distance $\frac{\sqrt{6}}{3}$ from this point.

बिन्दु (1, 2) से गुजरते हुए एक सरल रेखा किस दिशा में खींची जाए की सरल रेखा $x + y = 4$ के साथ इसके प्रतिच्छेद बिन्दु की बिन्दु (1, 2) से दूरी $\frac{\sqrt{6}}{3}$ रहे।

Ans. $\pi/12, 5\pi/12$

Sol. $\frac{x_1-1}{\cos\theta}=\frac{y_1-2}{\sin\theta}=\pm\frac{\sqrt{6}}{3}$ ($\because$ (1, 2) lie below the line)



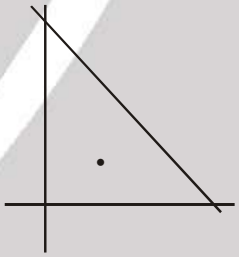
$\Rightarrow$ $x_1 = 1 + \frac{\sqrt{6}}{3}\cos\theta,\ y_1 = 2 + \frac{\sqrt{6}}{3}\sin\theta$

(x_1, y_1) lies on $x + y = 4 \Rightarrow 3 + \frac{\sqrt{6}}{3}(\sin\theta + \cos\theta) = 4$

$\sin\theta + \cos\theta = \frac{3}{\sqrt{6}} = \frac{3}{\sqrt{3}\cdot\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}(\sin\theta + \cos\theta) = \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$

$\sin(\theta+\frac{\pi}{4}) = \frac{\sqrt{3}}{2} = \sin 60^\circ \text{ or } \sin 120^\circ \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}$ $\left(\frac{-\sqrt{6}}{3} \text{ is rejected}\right)$

Hindi. $\frac{x_1-1}{\cos\theta}=\frac{y_1-2}{\sin\theta}=\pm\frac{\sqrt{6}}{3}$ ($\because$ (1, 2) रेखा के नीचे स्थित है)



$\Rightarrow$ $x_1 = 1 + \frac{\sqrt{6}}{3}\cos\theta,\ y_1 = 2 + \frac{\sqrt{6}}{3}\sin\theta$

(x_1, y_1) रेखा $x + y = 4$ पर स्थित है। $\Rightarrow 3 + \frac{\sqrt{6}}{3}(\sin\theta + \cos\theta) = 4$

$\sin\theta + \cos\theta = \frac{3}{\sqrt{6}} = \frac{3}{\sqrt{3}\cdot\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}(\sin\theta + \cos\theta) = \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$

$\sin(\theta+\frac{\pi}{4}) = \frac{\sqrt{3}}{2} = \sin 60^\circ \text{ or } \sin 120^\circ \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}$ $\left(\frac{-\sqrt{6}}{3} \text{ को नहीं लेना है}\right)$

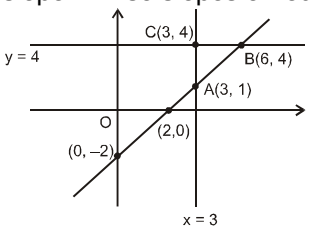
B-9. Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line $x – y = 2$. Find their equations and also find the area of triangle bounded by the three lines.

रेखा $x – y = 2$ पर 45° पर झुकी हुई तथा बिन्दु (3, 4) से गुजरने वाली दो रेखाऐं खींची जाती है, इन रेखाओं के समीकरण ज्ञात कीजिये। तीनों रेखाओं से परिबद्ध त्रिभुज का क्षेत्रफल ज्ञात करो।

Ans. x = 3, y = 4, 9/2 sq. units वर्ग इकाई

Sol. x – y = 2

slope = 1 so slopes of required lines are m = 0 and m = undefined



$\therefore$ lines are x = 3 and y – 4 = 0 given line is x – y = 2

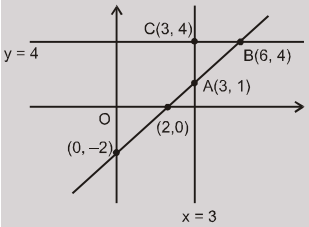
$\because$ ΔABC is right angle triangle so

area of $\Delta ABC = \frac{1}{2}$ × base × height

$\frac{1}{2} = \times (6-3) \times (4-1) = \frac{9}{2}$ sq. unit

Hindi. x – y = 2

प्रवणता = 1 इसलिए अभीष्ट रेखाओं की प्रवणताऐं m = 0 तथा m = अपरिभाषित



$\therefore$ रेखाऐं x = 3 तथा y – 4 = 0 दी गई रेखा x – y = 2

$\because$ ΔABC एक समकोण त्रिभुज है अतः

ΔABC का क्षेत्रफल = $\frac{1}{2}$ × आधार × ऊँचाई

= $\frac{1}{2} \times (6-3) \times (4-1) = \frac{9}{2}$ वर्ग इकाई

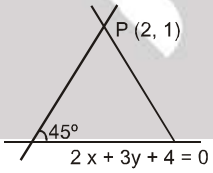
B-10. Find the equation of a straight lines which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line 2x + 3y + 4 = 0

उस रेखा का समीकरण जोकि बिन्दु (2, 1) से गुजरती है तथा रेखा 2x + 3y + 4 = 0 से $\pi/4$ का कोण बनाती हैं, है–

Ans. x – 5y + 3 = 0, 5x + y – 11 = 0

Sol. Let slope of required line is m

Now, y – 1 = m(x – 2)



$$\tan 45° = \left|\frac{m+\frac{2}{3}}{1-\frac{2m}{3}}\right| = \left|\frac{3m+2}{3-2m}\right| \quad \Rightarrow \quad \frac{3m+2}{3-2m} = \pm 1$$

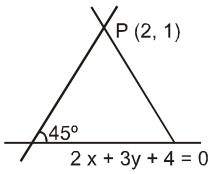
$\Rightarrow$ 3m + 2 = ± (3 – 2m) $\Rightarrow$ $m = \frac{1}{5}, -5$

Hence, $y - 1 = \frac{1}{5}(x-2) \Rightarrow x - 5y + 3 = 0$; $y - 1 = -5(x-2) \Rightarrow 5x + y - 11 = 0$

Hindi. माना अभीष्ट रेखा की प्रवणता m है।

तो, y – 1 = m(x – 2)

$$\tan 45° = \left|\frac{m+\frac{2}{3}}{1-\frac{2m}{3}}\right| = \left|\frac{3m+2}{3-2m}\right| \quad \Rightarrow \quad \frac{3m+2}{3-2m} = \pm 1$$



$\Rightarrow \quad 3m + 2 = \pm (3 - 2m) \quad \Rightarrow \quad m = \frac{1}{5}, -5$

अतः, $y - 1 = \frac{1}{5}(x - 2) \Rightarrow x - 5y + 3 = 0$; $y - 1 = -5(x - 2) \Rightarrow 5x + y - 11 = 0$

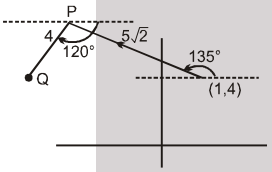
B-11. From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. Find co-ordinates of point Q.
आप (1, 4) से धनात्मक x-अक्ष के साथ 135° कोण से वामावर्त दिशा में $5\sqrt{2}$ इकाई दूरी चलकर धनात्मक x-अक्ष के साथ (दक्षिणावर्त दिशा में) 120° के कोण पर 4 इकाई दूरी से Q पर पहुँचता है। बिन्दु Q के निर्देशांक ज्ञात कीजिए।

Ans. $\left(-6, 9-2\sqrt{3}\right)$

Sol. For point P $\frac{x-1}{\cos 135°} = \frac{y-4}{\sin 135°} = \sqrt{2} \Rightarrow x = -4, y = 9$

For point Q

$$\frac{x+4}{\cos(-120°)} = \frac{y-9}{\sin(-120°)} = 4$$

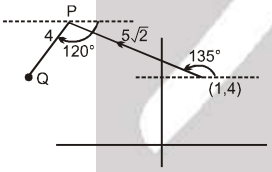


$x = -6, y = 9 - 2\sqrt{3}$

$\therefore \quad Q\left(-6, 9-2\sqrt{3}\right).$

Hindi बिन्दु P के लिए $\frac{x-1}{\cos 135°} = \frac{y-4}{\sin 135°} = \sqrt{2} \Rightarrow \quad x = -4, y = 9$

बिन्दु Q के लिए $\frac{x+4}{\cos(-120°)} = \frac{y-9}{\sin(-120°)} = 4$



$x = -6, y = 9 - 2\sqrt{3}$

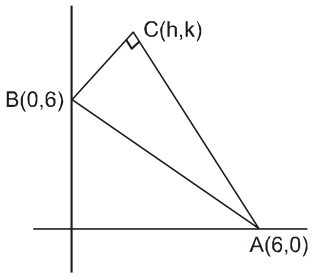
$\therefore \quad Q\left(-6, 9-2\sqrt{3}\right)$

B-12. The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6), then find the locus of third vertex of triangle.

समकोण त्रिभुज के कर्ण के सिरे (6, 0) तथा (0, 6) है, तब त्रिभुज के तीसरे शीर्ष का बिन्दुपथ ज्ञात कीजिए।

Ans. $x^2 + y^2 - 6x - 6y = 0$.

Sol. Slope of AC × slope of BC = – 1



AC की प्रवणता × BC की प्रवणता = – 1

$m_{AC} \times m_{Bc} = -1$

$\left(\frac{k-0}{h-6}\right)\left(\frac{k-6}{h-0}\right) = -1.$

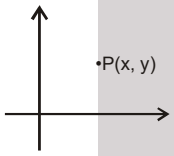
$k^2 - 6k = -h^2 + 6h$

$x^2 + y^2 - 6x - 6y = 0.$

B-13. A point moves in the x-y plane such that the sum of its distances from two mutually perpendicular lines is always equal to 3, then find the area enclosed by the locus of the point.
एक बिन्दु x-y समतल में इस प्रकार गमन करता है कि इसकी दो परस्पर लम्बवत रेखाओं से दूरियों का योग सदैव 3 इकाई रहता है। उस बिन्दु के बिन्दुपथ द्वारा परिबद्ध क्षेत्रफल है

Ans. 18

Sol.

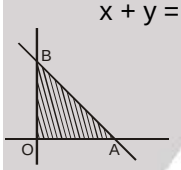


Let the point P be (x, y) $\Rightarrow$ $|x| + |y| = 3$ .....(i)

Case - 1 $x > 0, y > 0$

Equation (i) will become :

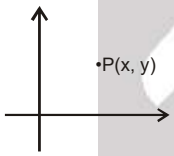
$x + y = 3$



Area (ΔOAB) = $\frac{9}{2}$

Similarly for each quadrant , a triangle will be formed. Hence area enclosed will be 18.

Hindi.

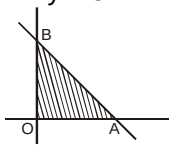


माना बिन्दु P (x, y) है। $\Rightarrow$ $|x| + |y| = 3$ .....(i)

स्थिति - 1 $x > 0, y > 0$

समीकरण (i)

$x + y = 3$



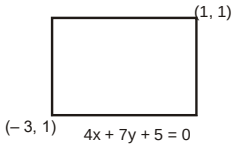
क्षेत्रफल (ΔOAB) = $\frac{9}{2}$

इसी प्रकार प्रत्येक चतुर्थांशं के लिए एक त्रिभुज बनेगा अतः परिबद्ध क्षेत्रफल 18 वर्ग इकाई होगा।

B-14. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are $(-3, 1)$ and $(1, 1)$. Then find the equations of other sides.
एक आयत की एक भुजा रेखा $4x + 7y + 5 = 0$ के अनुदिश है इस के दो शीर्ष $(-3, 1)$ तथा $(1, 1)$ है, तो अन्य भुजाओं का समीकरण ज्ञात करो।

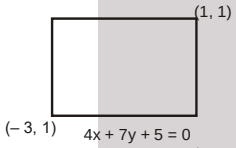
Ans. $7x - 4y + 25 = 0, 7x - 4y - 3 = 0, 4x + 7y = 11$

Sol.



Line $\perp$ to $4x + 7y + 5 = 0$ is $7x - 4y + \lambda = 0$
It passes through $(-3, 1)$ and $(1, 1)$
$-21 - 4 + \lambda = 0 \quad \Rightarrow \lambda = 25$
$7 - 4 + \lambda = 0 \quad \Rightarrow \lambda = -3$
Hence lines are $7x - 4y + 25 = 0, 7x - 4y - 3 = 0$
line $\parallel$ to $4x + 7y + 5 = 0$ passing through $(1, 1)$ is $4x + 7y + \lambda = 0$
$\Rightarrow \quad \lambda = -11 \quad \Rightarrow \quad 4x + 7y - 11 = 0$

Sol.



$4x + 7y + 5 = 0$ के लम्बवत् रेखा का समीकरण $7x - 4y + \lambda = 0$
यह बिन्दुओं $(-3, 1)$ एवं $(1, 1)$ से गुजरती है
$-21 - 4 + \lambda = 0 \Rightarrow \lambda = 25$
$7 - 4 + \lambda = 0 \Rightarrow \lambda = -3$
अतः रेखाऐं $7x - 4y + 25 = 0, 7x - 4y - 3 = 0$
$4x + 7y + 5 = 0$ के समान्तर एवं बिन्दु $(1, 1)$ से गुजरने वाली रेखा का समीकरण $4x + 7y + \lambda = 0$
$\Rightarrow \quad \lambda = -11 \quad \Rightarrow \quad 4x + 7y - 11 = 0$

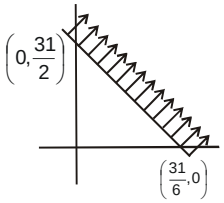
Section (C) : Position of point, linear inequation, perpendicular distance, image & foot, Area of Parallelogram

खण्ड (C) : बिन्दु की स्थिति, रैखिक असमिकाऐं लम्बवत् दूरी, प्रतिबिम्ब और लम्बपाद, समान्तर चतुर्भुज का क्षेत्रफल

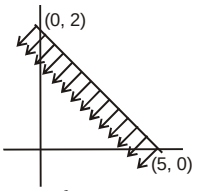
C-1. Plot the region
आरेख खीचींए–
(i) $6x + 2y \geq 31$ (ii) $2x + 5y \leq 10$ (iii) $8x + 3y + 6 > 0$ (iv) $x > 2$

Ans./Sol.

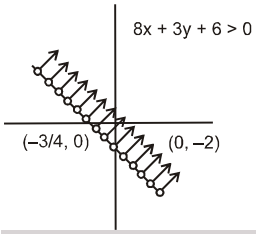
(i)



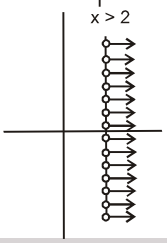
(ii)



(iii)



(iv)



C-2. Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line y = 3x – 4.
बिन्दू (2, 3) से रेखा y = 3x – 4 पर खींचे गऐ लम्ब की समीकरण, इस लम्ब के लम्बपाद के निर्देशांक तथा इस बिन्दु का रेखा के सापेक्ष प्रतिबिम्ब ज्ञात कीजिए।

Ans. Foot लम्बपाद $\left(\frac{23}{10}, \frac{29}{10}\right)$, Image प्रतिबिम्ब $\left(\frac{13}{5}, \frac{14}{5}\right)$, x + 3y – 11 = 0

Sol. foot of perpendicular $\frac{x-2}{3} = \frac{y-3}{-1} = -\frac{(3\times 2-3-4)}{3^2+(-1)^2} \Rightarrow (x, y) \equiv \left(\frac{23}{10}, \frac{29}{10}\right)$

image $\frac{x-2}{3} = \frac{y-3}{-1} = -2\frac{(3\times 2-3-4)}{3^2+(-1)^2} \Rightarrow (x, y) \equiv \left(\frac{13}{5}, \frac{14}{5}\right)$

slope of line perpendicular to the line y = 3x – 4 is $-\frac{1}{3}$ hence its equation

$y - 3 = -\frac{1}{3}(x - 2) \Rightarrow x + 3y - 11 = 0$

Hindi. लम्बपाद $\frac{x-2}{3} = \frac{y-3}{-1} = -\frac{(3\times 2-3-4)}{3^2+(-1)^2} \Rightarrow (x, y) \equiv \left(\frac{23}{10}, \frac{29}{10}\right)$

प्रतिबिम्ब $\frac{x-2}{3} = \frac{y-3}{-1} = -2\frac{(3\times 2-3-4)}{3^2+(-1)^2} \Rightarrow (x, y) \equiv \left(\frac{13}{5}, \frac{14}{5}\right)$

रेखा y = 3x – 4 के लम्ब्वत् रेखा की प्रवणता $-\frac{1}{3}$ है। अतः समीकरण है

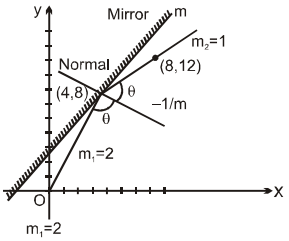
$y - 3 = -\frac{1}{3}(x - 2) \Rightarrow x + 3y - 11 = 0$

C-3. Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point A(4, 8) and reflected line passes through the point B (8, 12). Compute the slope of the mirror.
एक प्रकाश किरण, मूलबिन्दु से आती हुई एक रेखीय दर्पण से बिन्दु A(4, 8) पर टकराकर बिन्दु B (8, 12) से गुजरती है तब दर्पण रेखा की प्रवणता ज्ञात कीजिए।

Ans. $\frac{1+\sqrt{10}}{3}$

Sol. Let the slope of the line mirror is m. Hence slope of normal is $\frac{-1}{m}$.

माना रेखीय दर्पण की प्रवणता m है अतः अभिलम्ब की प्रवणता $\frac{-1}{m}$ है।



Equating the two values of θ, we get θ के दो मानों की तुलना करने पर

$$\frac{-\frac{1}{m}-2}{1-\frac{2}{m}} = \frac{1+\frac{1}{m}}{1-\frac{1}{m}} \qquad \Rightarrow \qquad -\frac{2m+1}{m-2} = \left(\frac{m+1}{m-1}\right)$$

$3m^2 - 2m - 3 = 0$

$\therefore$ $m = \frac{1+\sqrt{10}}{3}$ or $m = \frac{1+\sqrt{10}}{3}$ which is rejected (think !) जो की अस्वीकार्य है।

$[m \in (1, 2)]$

$\therefore$ Slope of the miror is $\frac{1+\sqrt{10}}{3}$. $\therefore$ दर्पण की प्रवणता $\frac{1+\sqrt{10}}{3}$ है।.

C-4. Find the nearest point on the line $3x + 4y - 1 = 0$ from the origin.
मूलबिन्दु से रेखा $3x + 4y - 1 = 0$ के नजदीक का बिन्दु है।

Ans. $\left(\frac{3}{25}, \frac{4}{25}\right)$

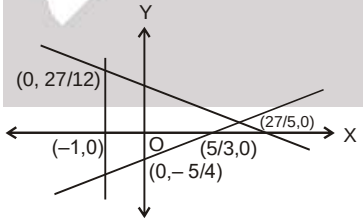
Sol. Required point is foot of perpendicular from (0, 0) on the given line which is $\frac{\alpha-0}{3} = \frac{\beta-0}{4} = \frac{-(-1)}{25}$

अभीष्ट बिन्दु (0, 0) से दी गई रेखा पर डाले गये लम्ब का पाद है जो $\frac{\alpha-0}{3} = \frac{\beta-0}{4} = \frac{-(-1)}{25}$ है।

C-5. Find the position of the origin with respect to the triangle whose sides are $x + 1 = 0$, $3x - 4y - 5 = 0$, $5x + 12y - 27 = 0$.
मूल बिन्दु की रेखाओं $x + 1 = 0$, $3x - 4y - 5 = 0$ व $5x + 12y - 27 = 0$ से बनने वाले त्रिभुज के सापेक्ष स्थिति ज्ञात करो।

Ans. Inside अंदर स्थित है।

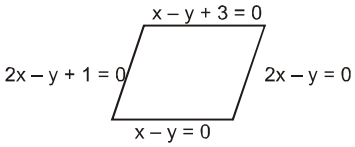
Sol. Plot diagram आरेख खीचिएं



C-6. Find the area of parallelogram whose two sides are $y = x + 3$, $2x - y + 1 = 0$ and remaining two sides are passing through (0, 0).
उस समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए जिसकी दो भुजाएँ $y = x + 3$, $2x - y + 1 = 0$ है तथा अन्य दो भुजायें (0, 0) से गुजरती है।

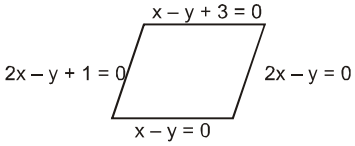
Ans. 3 sq. unit वर्ग इकाई

Sol.



Area of parallogram = $\left|\frac{(3-0)(1-0)}{(2-1)}\right|$ = 3

Hindi.



समान्तर चतुर्भुज का क्षेत्रफल = $\left|\frac{(3-0)(1-0)}{(2-1)}\right|$ = 3

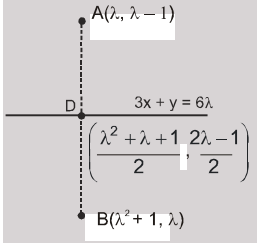
C-7. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so, find λ.

क्या λ का कोई वास्तविक मान है जिसके लिए बिन्दु $(\lambda, \lambda - 1)$ का रेखा दर्पण $3x + y = 6\lambda$ के सापेक्ष प्रतिबिम्ब बिन्दु $(\lambda^2 + 1, \lambda)$ हो ? यदि ऐसा हो, तो λ का मान ज्ञात कीजिए।

Ans. 2

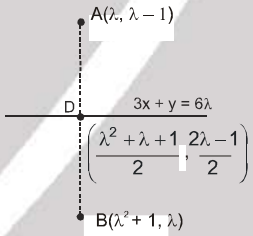
Sol. D is mid point of AB and lies on the line $3x + y = 6\lambda$

$\Rightarrow \quad 3 \cdot \frac{\lambda^2+\lambda+1}{2} + \frac{2\lambda-1}{2} = 6\lambda$



$3\lambda^2 - 7\lambda + 2 = 0$ .....(1)

$\lambda = \frac{1}{3}, 2$



multiplication of slope of AB & line = –1

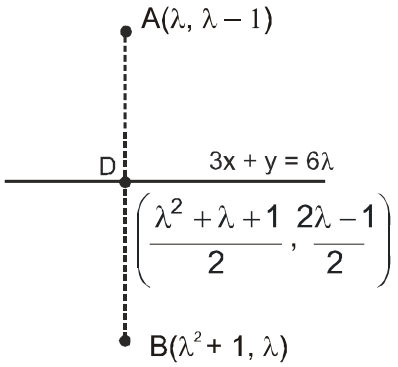
$\frac{-1}{\lambda-\lambda^2-1}(-3) = -1 \quad \Rightarrow \quad \lambda^2 - \lambda - 2 = 0$ .....(2)

$\lambda = -1, 2 \quad \Rightarrow \quad \lambda = 2$ satisfies both (1) & (2)

Hindi.

AB का मध्य बिन्दु D रेखा $3x + y = 6\lambda$ पर स्थित है।

अतः $3 \cdot \frac{\lambda^2+\lambda+1}{2} + \frac{2\lambda-1}{2} = 6\lambda$



$3\lambda^2 - 7\lambda + 2 = 0$ ...(1)

$\lambda = \frac{1}{3}, 2$

रेखा AB और दी गई रेखा परस्पर लम्बवत् है अतः
इन रेखाओं की प्रवणताओं का गुणनफल = –1

$\frac{-1}{\lambda - \lambda^2 - 1}(-3) = -1 \quad \Rightarrow \quad \lambda^2 - \lambda - 2 = 0$ ... (2)

$\lambda = -1, 2$ अतः $\lambda = 2$ अभीष्ट हल है।

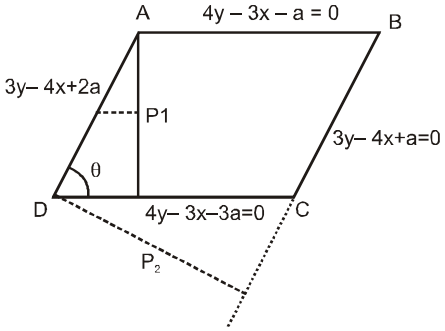
C-8. Find the equations of two straight lines which are parallel to x + 7y + 2 = 0 and at $\sqrt{2}$ distance away from it.
सरल रेखा x + 7y + 2 = 0 के समान्तर सरल रेखाओं के समीकरण है, जो इससे $\sqrt{2}$ दुरी पर है।
Ans. x + 7y + 12 = 0, x + 7y – 8 = 0

Sol. Let line be $x + 7y + \lambda = 0 \Rightarrow$ since distance of this lines $\left|\frac{2-\lambda}{\sqrt{50}}\right| = \sqrt{2} \Rightarrow \lambda = 12, -8.$

Hindi. माना रेखा $x + 7y + \lambda = 0 \Rightarrow$ इस रेखा की दुरी $\left|\frac{2-\lambda}{\sqrt{50}}\right| = \sqrt{2} \Rightarrow \lambda = 12, -8.$

C-9. Prove that the area of the parallelogram contained by the lines 4y – 3x – a = 0, 3y – 4x + a = 0, 4y – 3x – 3a = 0 and 3y – 4x + 2a = 0 is $\frac{2}{7}a^2$.
सिद्ध कीजिए कि रेखाओं 4y – 3x – a = 0, 3y – 4x + a = 0, 4y – 3x – 3a = 0 एवं 3y – 4x + 2a = 0 से बने समान्तर चतुर्भुज का क्षेत्रफल $\frac{2}{7}a^2$ वर्ग इकाई है।

Sol. $\tan\theta = \left|\frac{\frac{4}{3}-\frac{3}{4}}{1+\frac{4}{3}\cdot\frac{3}{4}}\right| = \left|\frac{\frac{16-9}{12}}{1+1}\right| \quad \frac{7}{24} \quad \Rightarrow \quad \text{cosec}\,\theta = \frac{25}{7}$

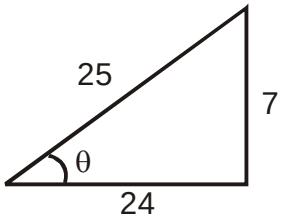


P_1 is ⊥ from AB to CD, P_2 is ⊥ from AD to BC
for finding P_1 choose arbitrary point (a, a) on AB

$\therefore \quad P_1 = \dfrac{|4a - 3a - 3a|}{5} = \dfrac{2a}{5}$

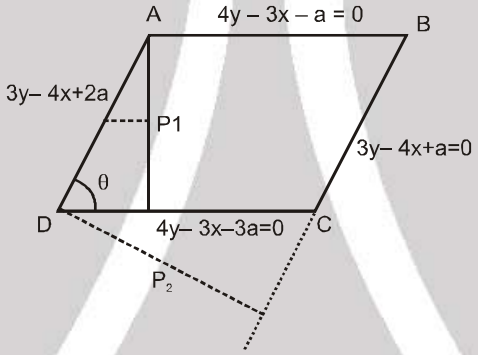
for P_2 choose arbitrary point (a/2, 0) on AD

$\therefore \quad P_2 = \dfrac{|0 - 2a + a|}{5} = \dfrac{a}{5}$



$\because$ Area = P_1P_2 cosec $\theta = \dfrac{2a^2}{7}$

Hindi. $\tan\theta = \left|\dfrac{\frac{4}{3} - \frac{3}{4}}{1 + \frac{4}{3}\cdot\frac{3}{4}}\right| = \left|\dfrac{\frac{16-9}{12}}{1+1}\right| \dfrac{7}{24} \quad \Rightarrow \quad \text{cosec}\,\theta = \dfrac{25}{7}$



P_1 रेखाओं AB और CD के बीच की लम्बवत् दूरी है

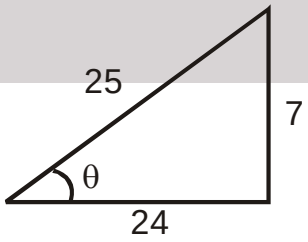
तथा P_2 रेखाओं AD और BC के बीच की लम्बवत् दूरी है।

P_1 ज्ञात करने के लिए मानाकि AB पर कोई स्वेच्छ बिन्दु (a, a) है।

$\therefore \quad P_1 = \dfrac{|4a - 3a - 3a|}{5} = \dfrac{2a}{5}$

P_2 ज्ञात करने के लिए मानाकि AD पर कोई स्वेच्छ बिन्दु (a/2, 0) है।

$\therefore \quad P_2 = \dfrac{|0 - 2a + a|}{5} = \dfrac{a}{5}$



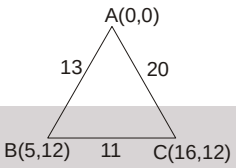
$\because$ क्षेत्रफल = P_1P_2 cosec $\theta = \dfrac{2a^2}{7}$

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus
खण्ड (D) : केन्द्रक लम्बकेन्द्र, परिकेन्द्र बहिष्केन्द्र, बिन्दुपथ

D-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of
(i) Centroid (ii) Circumcentre
(iii) Incentre (iv) Excentre opposite to vertex (5, 12)
शीर्षो (0, 0), (5, 12) व (16, 12) वाले त्रिभुज के लिये निम्न ज्ञात करो
(i) केन्द्रक (ii) परिकेन्द्र
(iii) अन्तः केन्द्र (iv) शीर्ष (5, 12) के सम्मुख बर्हिकेन्द्र

Ans. (i) (7, 8) (ii) $\left(\frac{21}{2}, \frac{8}{3}\right)$ (iii) (7, 9) (iv) (27, –21)

Sol. (i) centroid $\left(\frac{0+5+16}{3}, \frac{0+12+12}{3}\right) \equiv (7, 8)$



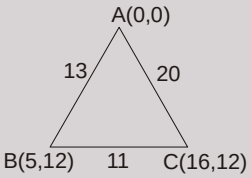
(ii) Let coordinates of circumcentre is O (x, y).
Therefore OA = OB = OC $\Rightarrow x^2 + y^2 = (x-5)^2 + (y-12)^2 = (x-16)^2 + (y-12)^2$
$\Rightarrow$ $x^2 + y^2 = (x-5)^2 + (y-12)^2 \Rightarrow 10x + 24y = 16g$
$(x-5)^2 + (y-12)^2 = (x-16)^2 + (y-12)^2 \Rightarrow 2x = 21 \Rightarrow x = \frac{21}{2}, y = \frac{8}{3}$

(iii) $I\left(\frac{0\times11+5\times20+16\times13}{13+20+11}, \frac{0\times11+12\times20+13\times12}{13+20+11}\right) \equiv (7, 9)$

(iv) $I^2 = \left(\frac{-5\times20+13\times16+11\times13}{-20+13+11}, \frac{-12\times20+0\times11+13\times12}{-20+13+11}\right) \equiv (27, -21)$

Hindi (i) केन्द्रक $\left(\frac{0+5+16}{3}, \frac{0+12+12}{3}\right) \equiv (7, 8)$

(ii) माना परिकेन्द्र के निर्देशांक O (x, y) है
इसलिए OA = OB = OC

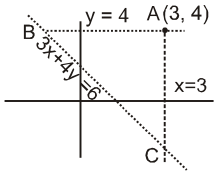


$\Rightarrow$ $x^2 + y^2 = (x-5)^2 + (y-12)^2 = (x-16)^2 + (y-12)^2$
$\Rightarrow$ $x^2 + y^2 = (x-5)^2 + (y-12)^2 \Rightarrow 10x + 24y = 16g$
$(x-5)^2 + (y-12)^2 = (x-16)^2 + (y-12)^2 \Rightarrow 2x = 21 \Rightarrow x = \frac{21}{2}, y = \frac{8}{3}$

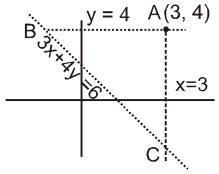
(iii) $I\left(\frac{0\times11+5\times20+16\times13}{13+20+11}, \frac{0\times11+12\times20+13\times12}{13+20+11}\right) \equiv (7, 9)$

(iv) $I^2 = \left(\frac{-5\times20+13\times16+11\times13}{-20+13+11}, \frac{-12\times20+0\times11+13\times12}{-20+13+11}\right) \equiv (27, -21)$

D-2. Find the sum of coordinates of the orthocentre of the triangle whose sides are x = 3, y = 4 and 3x + 4y = 6.
एक त्रिभुज की भुजाएँ x = 3, y = 4 एवं 3x + 4y = 6 है, तो त्रिभुज के लम्बकेन्द्र के निर्देशांक का योगफल है –
Ans. 7



Sol. From figure it is clear that A is orthocentre of ΔABC



Hindi. चित्र से स्पष्ट है कि A लम्बकेन्द्र है।

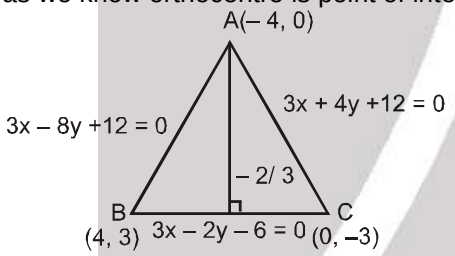
D-3. Find equations of altitudes and the co-ordinates of the othocentre of the triangle whose sides are $3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$.
भुजाओं $3x - 2y = 6$, $3x + 4y + 12 = 0$ व $3x - 8y + 12 = 0$ वाले त्रिभुज के शीर्षलम्बों के समीकरण तथा लम्ब केंद्र के निर्देशांक ज्ञात करो।

Ans. $2x + 3y + 8 = 0$, $4x - 3y - 7 = 0$, $8x + 3y + 9 = 0$, orthocentre लम्बकेंद्र $\left(-\frac{1}{6}, -\frac{23}{9}\right)$.

Sol. Altitudes passing through A is $y - 0 = -(x + 4)$

$y - 0 = \left(\frac{-2}{3}\right)(x + 4) \quad \Rightarrow \quad 3y = -2x - 8 \quad \Rightarrow \quad 2x + 3y + 8 = 0$

similarly equation of altitudes passing through other vertex are $4x - 3y - 7 = 0$ and $8x + 3y + 9 = 0$
as we know orthocentre is point of intersection of altitudes

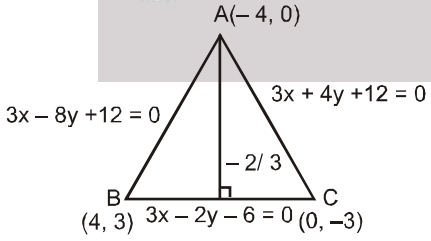


Hence its coordinates is $\left(-\frac{1}{6}, \frac{-23}{9}\right)$

Hindi. A से गुजरने वाले शीर्ष लम्ब का समीकरण $y - 0 = -(x + 4)$

$y - 0 = \left(\frac{-2}{3}\right)(x + 4) \quad \Rightarrow \quad 3y = -2x - 8 \quad \Rightarrow \quad 2x + 3y + 8 = 0$

इसी प्रकार अन्य शीर्षों से गुजरने वाले शीर्ष लम्बों के समीकरण $4x - 3y - 7 = 0$ तथा $8x + 3y + 9 = 0$ होंगे।
जैसा कि हम जानते है कि लम्बकेन्द्र शीर्ष लम्बों का प्रतिच्छेद बिन्दु होता है।



अतः इसके निर्देशांक $\left(-\frac{1}{6}, \frac{-23}{9}\right)$ होगें।

D-4. A triangle has the lines $y = m_1x$ and $y = m_2x$ for two of its sides, where m_1, m_2 are the roots of the equation $x^2 + ax - 1 = 0$, then find the orthocentre of triangle.
एक त्रिभुज की दो भुजाएं $y = m_1x$ तथा $y = m_2x$ है जहाँ m_1, m_2 समीकरण $x^2 + ax - 1 = 0$ के मूल है। तब त्रिभुज का लम्बकेन्द्र है।

Ans. (0, 0)

Sol. m_1, m_2 are roots of the equation $x^2 + ax - 1 = 0$. $\therefore$ $m_1m_2 = -1$.
These sides are perpendicular to each other, so intersection point of these sides $y = m_1x$ and $y = m_2x$ is origin which is locus of Δ.

Hindi. m_1, m_2 समीकरण $x^2 + ax - 1 = 0$ के मूल है $\therefore$ $m_1m_2 = -1$.
ये भुजाऐं एक दूसरे लम्बवत् है। इसलिये इन भुजाओं $y = m_1x$ और $y = m_2x$ का प्रतिच्छेद बिन्दु मूल बिन्दु है जोकि Δ का बिन्दुपथ है।

D-5. Prove that the circumcentre, orthocentre, incentre & centroid of the triangle formed by the points A(–1, 11); B (–9, –8); C (15, –2) are collinear, without actually finding any of them.
सिद्ध कीजिए बिन्दुओं A(–1, 11); B (–9, –8); C (15, –2) से बनने वाले त्रिभुज का परिकेन्द्र, लम्बकेन्द्र, अन्तःकेन्द्र एवं केन्द्रक संरेखीय है। (किसी को भी ज्ञात किए बिना।)

Sol. $AB = \sqrt{64+361} = \sqrt{425}$; $BC = \sqrt{576+36} = \sqrt{612}$
$AC = \sqrt{256+169} = \sqrt{425}$; $AB = AC \neq BC$,
$\therefore$ triangle is isosceles and in isosceles triangle O, H, I, G are collinear

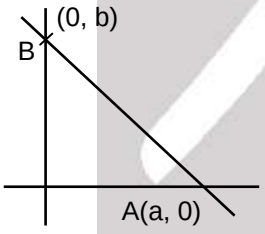
Hindi. $AB = \sqrt{64+361} = \sqrt{425}$; $BC = \sqrt{576+36} = \sqrt{612}$
$AC = \sqrt{256+169} = \sqrt{425}$; $AB = AC \neq BC$
अतः त्रिभुज ABC समद्विबाहु त्रिभुज है और एक समद्विबाहु त्रिभुज में O, H, I, G संरेखीय होते है।

D-6. Find locus of centroid of ΔAOB if line AB passes through (3, 2), A and B are on coordinate axes.

ΔAOB के केन्द्रक का बिन्दुपथ ज्ञात कीजिए यदि रेखा AB, बिन्दु (3, 2), से गुजरती है तथा A और B निर्देशांक अक्षों पर है –

Ans. $\frac{3}{x}+\frac{2}{y}=3$

Sol.



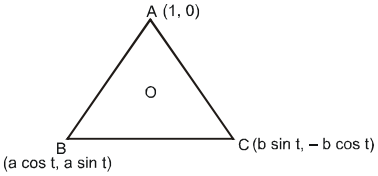
$h=\frac{a}{3}, k=\frac{b}{3} \Rightarrow \frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{3}{a}+\frac{2}{b}=1 \Rightarrow \frac{3}{3h}+\frac{2}{3k}=1$

$\frac{3}{x}+\frac{2}{y}=3$

D-7. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1, 0), where 't' is the parameter.
एक त्रिभुज के केन्द्रक का बिन्दुपथ ज्ञात कीजिए जिसके शीर्षों के निर्देशांक (a cos t, a sin t), (b sin t, –b cos t) और (1, 0) हैं, जहाँ 't' प्राचल है।

Ans. $(3x - 1)^2 + 9y^2 = a^2 + b^2$

Sol.



Let co - ordinates of centroid be (x_1, y_1) मानाकि केन्द्रक के निर्देशांक (x_1, y_1) है

$\Rightarrow \quad x_1 = \frac{1+a\cos t+b\sin t}{3} \quad \Rightarrow \quad a\cos t + b\sin t = 3x_1 - 1 \quad ... (1)$

and (तथा) $y_1 = \frac{0+a\sin t-b\cos t}{3} \quad \Rightarrow \quad a\sin t - b\cos t = 3y_1 \quad ... (2)$

square (1), (2) and add them to get (1) व (2) को वर्गकर जोड़ने पर

$a^2 + b^2 = (3x_1 - 1)^2 + (3y_1)^2$

$\therefore$ Locus of centroid (x_1, y_1) is अतः केन्द्रक (x_1, y_1) का बिन्दुपथ है

$\Rightarrow \quad (3x - 1)^2 + 9y^2 = a^2 + b^2$

D-8. Show that equation of the locus of a point which moves so that difference of its distance from two given points (ae, 0) and (–ae, 0) is equal to 2a is $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$.

सिद्ध कीजिए एक ऐसे बिन्दु का बिन्दुपथ जिसकी दो स्थिर बिन्दुओं (ae, 0) व (–ae, 0) से दूरियों का अंतर सदैव स्थिर तथा 2a के बराबर होता है, $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$ है।

Sol. Let point is P(x, y) and A(ae, 0) and B(–ae,0)

Given $|PA - PB| = 2a \quad \Rightarrow \quad \left|\sqrt{(x-ae)^2+y^2} - \sqrt{(x+ae)^2+y^2}\right| = 2a$

Let $\sqrt{(x-ae)^2+y^2} = A$, $\sqrt{(x+ae)^2+y^2} = B$. Hence $A - B = 2a$

$A^2 - B^2 = (A + B)(A - B) \quad \Rightarrow \quad A + B = -2xe$. Hence $2A = 2a - 2xe \Rightarrow A = a - ex$

$(x - ae)^2 + y^2 = (a - ex)^2 \quad \Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$

Hindi माना बिन्दु P(x, y) है तथा A(ae, 0) और B(–ae,0) है

दिया गया है $|PA - PB| = 2a \quad \Rightarrow \quad \left|\sqrt{(x-ae)^2+y^2} - \sqrt{(x+ae)^2+y^2}\right| = 2a$

Let $\sqrt{(x-ae)^2+y^2} = A$, $\sqrt{(x+ae)^2+y^2} = B$. Hence $A - B = 2a$

$A^2 - B^2 = (A + B)(A - B) \quad \Rightarrow \quad A + B = -2xe$. Hence $2A = 2a - 2xe \Rightarrow \quad A = a - ex$

$(x - ae)^2 + y^2 = (a - ex)^2 \quad \Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$

D-9. Find the locus of point of intersection of the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$, where α is a parameter.

रेखाओं $x\cos\alpha + y\sin\alpha = a$ और $x\sin\alpha - y\cos\alpha = b$, जहाँ α एक प्राचल है, के प्रतिच्छेद बिन्दु का बिन्दुपथ ज्ञात कीजिए।

Ans. $x^2 + y^2 = a^2 + b^2$

Sol Given $x\cos\alpha + y\sin\alpha = a$ .....(1) $\quad x\sin\alpha - y\cos\alpha = b$ .....(2)

square (1) and (2) then add them $x^2 + y^2 = a^2 + b^2$

Hindi. दिया गया है $x\cos\alpha + y\sin\alpha = a$ .....(1) $\quad x\sin\alpha - y\cos\alpha = b$ .....(2)

(1) और (2) का वर्ग करके जोड़ने पर $x^2 + y^2 = a^2 + b^2$

D-10. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P.

x–अक्ष के समान्तर एवं P(h, k) से गुजरने वाली रेखा तथा रेखाओं y = x एवं x + y = 2 द्वारा निर्मित त्रिभुज का क्षेत्रफल $4h^2$ हैं, तो बिन्दु P का बिन्दु पथ ज्ञात कीजिए।

Ans. y = 2x + 1 or y = –2x + 1

Sol. A line passing through P(h, k) and parallel to x-axis is y = k. .....(i)

The other lines given are y = x .....(ii)

and y + x = 2 .....(iii)

Let ABC be the Δ formed by the points of intersection of the lines (i), (ii) and (iii).

$\therefore$ A(k, k) , B(1, 1), C (2–k, k)

$\therefore$ Area of Δ ABC $= \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$

$C_1 \to C_1 - C_2$ $\quad \frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$

$\Rightarrow \quad \frac{1}{2}|(2-2k)(k-1)| = 4h^2 \quad \Rightarrow \quad (k-1)^2 = 4h^2$

$\Rightarrow \quad k - 1 = 2h, \quad k - 1 = -2h \quad \Rightarrow \quad k = 2h + 1 \quad k = -2h + 1$

$\therefore$ locus of (h, k) is y = 2x + 1 y = – 2x + 1

Hindi. बिन्दु P(h, k) से गुजरने वाली तथा x-अक्ष के समान्तर रेखा है y = k .....(i)

अन्य दी गयी रेखायें y = x .....(ii)

तथा y + x = 2 .....(iii)

मानाकि (i) , (ii) व (iii) के प्रतिच्छेदी बिन्दु से बनने वाला त्रिभुज ABC है तब A(k, k) , B(1, 1), C (2–k, k)

$\therefore$ Δ ABC का क्षेत्रफल $= \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$

$C_1 \to C_1 - C_2$ $\quad \frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$

$\Rightarrow \quad \frac{1}{2}|(2-2k)(k-1)| = 4h^2 \quad \Rightarrow \quad (k-1)^2 = 4h^2$

$\Rightarrow$ k – 1 = 2h, k – 1 = – 2h $\Rightarrow$ k = 2h + 1 k = – 2h + 1

अतः (h, k) का बिन्दुपथ y = 2x + 1, y = – 2x + 1 है।

Section (E) : Angle Bisector, condition of concurrency, family of straight lines
खण्ड (E) : कोण अर्द्धक, संगामी होने का प्रतिबन्ध, सरल रेखाओं का निकाय

E-1. Find equations of acute and obtuse angle bisectors of the angle between the lines 4x + 3y – 7 = 0 and 24x + 7y – 31 = 0.

रेखाओं 4x + 3y – 7 = 0 तथा 24x + 7y – 31 = 0 के मध्य न्यून कोण व अधिक कोण समद्विभाजकों के समीकरण ज्ञात करो।

Ans. acute 2x + y – 3 = 0, obtuse x – 2y + 1 = 0, origin lies in obtuse angle bisector.

Ans. न्यून कोण अर्द्धक 2x + y – 3 = 0, अधिक कोण अर्धक x – 2y + 1 = 0, मूल बिंदू अधिक कोण में है।

Sol. L_1 : 4x + 3y – 7 = 0 ; L_2 : 24x + 7y – 31 = 0

$a_1 a_2 + b_1 b_2 = 4 \times 24 + 3 \times 7 > 0$

Hence + sign gives obtuse angle bisector and – sign gives acute angle bisector

Now, put origin in both $4 \times 0 + 3 \times 0 - 7 < 0 \quad \Rightarrow \quad 24 \times 0 + 7(0) - 31 < 0$

Hence $\oplus$ sign gives that bisector in which origin lies. Hence origin lies in obtuse angle bisector

Now, equation of bisector $\left(\frac{4x+3y-7}{5}\right) = \pm\left(\frac{24x+7y-31}{25}\right)$

$\Rightarrow$ + sign $x - 2y + 1 = 0$ $\Rightarrow$ – sign $2x + y - 3 = 0$

Hindi. $L_1 : 4x + 3y - 7 = 0$; $L_2 : 24x + 7y - 31 = 0$

$a_1 a_2 + b_1 b_2 = 4 \times 24 + 3 \times 7 > 0$

अतः + चिन्ह लेने पर अधिक कोण अर्द्धक तथा – चिन्ह लेने पर न्यून कोण अर्द्धक प्राप्त होगा।

अब मूलबिन्दु को दोनो में रखने पर $4 \times 0 + 3 \times 0 - 7 < 0$ $\Rightarrow$ $24 \times 0 + 7(0) - 31 < 0$

अतः + चिन्ह रखने वाला अर्द्धक में मूल बिन्दु स्थित है। अतः मूलबिन्दु अधिक कोण अर्द्धक में स्थित है।

अतः अर्द्धक का समीकरण $\left(\frac{4x+3y-7}{5}\right) = \pm\left(\frac{24x+7y-31}{25}\right)$

$\Rightarrow$ + चिन्ह $x - 2y + 1 = 0$ $\Rightarrow$ – चिन्ह $2x + y - 3 = 0$

E-2. Find the equation of a straight line passing through the point (4, 5) and equally inclined to the lines $3x = 4y + 7$ and $5y = 12x + 6$.

बिन्दु (4, 5) से गुजरने वाली एवं रेखाओं $3x = 4y + 7$ तथा $5y = 12x + 6$ से समान कोण पर झुकी रेखा की समीकरण ज्ञात कीजिए।

Ans. $9x - 7y = 1$, $7x + 9y = 73$

Sol. The lines will pass through (4, 5) & parallel to the bisectors between them $\frac{3x-4y-7}{5} = \pm\frac{12x-5y+6}{13}$

by taking + sign, we get $21x + 27y + 121 = 0$. Now by taking – sign, we get $99x - 77y - 61 = 0$

so slopes of bisectors are $-\frac{7}{9}, \frac{9}{7}$

Equation of lines are $y - 5 = \frac{-7}{9}(x - 4)$ and $y - 5 = \frac{9}{7}(x - 4)$

$\Rightarrow$ $7x + 9y = 73$ and $9x - 7y = 1$

Hindi. रेखाऐं (4,5) से गुजरेगी तथा उनके कोण अर्धकों के समान्तर होगी them $\frac{3x-4y-7}{5} = \pm\frac{12x-5y+6}{13}$

धनात्मक चिन्ह लेने पर $21x + 27y + 121 = 0$. ऋणात्मक चिन्ह लेने पर $99x - 77y - 61 = 0$

अतः कोण अर्धकों का ढाल होंगे ,

रेखाओं के समीकरण होंगे $y - 5 = \frac{-7}{9}(x - 4)$ तथा $y - 5 = \frac{9}{7}(x - 4)$

$\Rightarrow$ $7x + 9y = 73$ तथा $9x - 7y = 1$

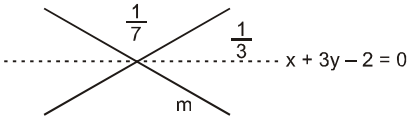
E-3. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$,then find equation of other line

रेखा $x + 3y - 2 = 0$ एक सरल रेखा युग्म के मध्य कोण को समद्विभाजित करती हैं। यदि रेखा युग्म की एक समीकरण $x - 7y + 5 = 0$ हो, तो अन्य रेखा की समीकरण है –

Ans. $5x + 5y - 3 = 0$

Sol. Point of intersection of $x + 3y - 2 = 0$ and $x - 7y + 5 = 0$ is $\left(-\frac{1}{10}, \frac{7}{10}\right)$

$$\left(\frac{-\frac{1}{3}-m}{1-\frac{m}{3}}\right) = -\left(\frac{-\frac{1}{3}-\frac{1}{7}}{1-\frac{1}{21}}\right)$$



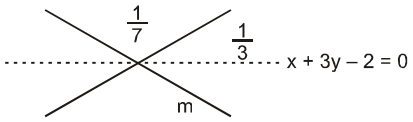
$\Rightarrow$ $\frac{-1-3m}{3-m} = \frac{10}{20} = \frac{1}{2}$ $\Rightarrow$ $-2 - 6m = 3 - m$ $\Rightarrow$ $m = -1$

Hence requred equation

$y - \frac{7}{10} = -1\left(x + \frac{1}{10}\right)$ $\Rightarrow$ $10y - 7 = -10x - 1$ $\Rightarrow$ $10x + 10y = 6 \Rightarrow 5x + 5y = 3$

Hindi. $x + 3y - 2 = 0$ एवं $x - 7y + 5 = 0$ का प्रतिच्छेद बिन्दु $\left(-\frac{1}{10}, \frac{7}{10}\right)$ होगा

$$\left(\frac{-\frac{1}{3}-m}{1-\frac{m}{3}}\right) = -\left(\frac{-\frac{1}{3}-\frac{1}{7}}{1-\frac{1}{21}}\right)$$



$\Rightarrow \quad \frac{-1-3m}{3-m} = \frac{10}{20} = \frac{1}{2} \quad -2 - 6m = 3 - m \quad \Rightarrow \quad m = -1$

अतः अभीष्ट समीकरण

$y - \frac{7}{10} = -1\left(x + \frac{1}{10}\right) \quad \Rightarrow \quad 10y - 7 = -10x - 1 \quad \Rightarrow \quad 10x + 10y = 6 \Rightarrow 5x + 5y = 3$

E-4. Find the value of λ such that lines $x + 2y = 3$, $3x - y = 1$ and $\lambda x + y = 2$ can not form a triangle.
λ के मान ज्ञात कीजिए जबकि रेखाएँ $x + 2y = 3$, $3x - y = 1$ तथा $\lambda x + y = 2$ त्रिभुज नहीं बना सकती है।

Ans. $\frac{6}{5}, \frac{1}{2}, -3$

Sol. Line are concurrent so रेखाएं संगामी है इसलिए $\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 1 \\ \lambda & 1 & 2 \end{vmatrix} = 0 \Rightarrow 5\lambda - 6 = 0 \Rightarrow \lambda = \frac{6}{5}$.

When two lines are parallel then $\lambda = \frac{1}{2}, -3$. जब दो रेखाऐं समान्तर हो, तब $\lambda = \frac{1}{2}, -3$. **Ans.**

E-5. Find values of λ for which line $y = x + 1$, $y = \lambda x + 2$ and $y = (\lambda^2 + \lambda - 1)x + 3$ are con-current.

λ के मान ज्ञात कीजिए जिसके लिए $y = x + 1$, $y = \lambda x + 2$ और $y = (\lambda^2 + \lambda - 1)x + 3$ संगामी है।

Ans. 0

Sol. $\begin{vmatrix} \lambda^2 + \lambda - 1 & \lambda & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$(\lambda^2 + \lambda - 1)\,1 - \lambda(2) + 1 = 0$

$\lambda^2 - \lambda = 0 \quad \Rightarrow \quad \lambda = 0, 1$

but when $\lambda = 1$ all the three lines are parallel so only possible value λ is 0

परन्तु $\lambda = 1$ तब सभी तीन रेखाऐं समान्तर है अतः λ का समान्तर मान 0 है।

E-6. Find the equation to the straight line passing through
(i) The point (3, 2) and the point of intersection of the lines $2x + 3y = 1$ and $3x - 4y = 6$.
(ii) The intersection of the lines $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and perpendicular to the straight line $y - x = 8$.
उस सरल रेखा का समीकरण ज्ञात करो जो
(i) बिन्दु (3, 2) तथा रेखाओं $2x + 3y = 1$ व $3x - 4y = 6$ के प्रतिच्छेदन बिन्दू से गुजरती है।
(ii) रेखाओं $x + 2y + 3 = 0$ व $3x + 4y + 7 = 0$ के प्रतिच्छेदन बिन्दु से गुजरती है तथा सरल रेखा $y - x = 8$ के लम्बवत् है।

Ans. (i) $43x - 29y = 71$ (ii) $x + y + 2 = 0$

Sol.

(i) By family of lines $(2x + 3y – 1) + \lambda (3x – 4y – 6) = 0$ it passes through (3, 2)

$(6 + 6 – 1) + \lambda (9 – 8 – 6) = 0 \quad \Rightarrow \quad \lambda = \frac{11}{5}$

$(2x + 3y – 1) + \frac{11}{5} (3x – 4y – 6) = 0 \quad \Rightarrow \quad 43x – 29y = 71$

(ii) By family of lines $(x + 2y + 3) + \lambda (3x + 4y + 7) = 0$

This is $\perp$ to $x – y + 8 = 0$

Hence $m_1m_2 = – 1 \quad \Rightarrow \quad – \frac{(3\lambda + 1)}{(4\lambda + 2)} (1) = – 1 \quad \Rightarrow \quad 3\lambda + 1 = 4\lambda + 2$

$\Rightarrow \quad \lambda = – 1 \quad \Rightarrow \quad (x + 2y + 3) – (3x + 4y + 7) = 0$

$\Rightarrow \quad –2x – 2y – 4 = 0 \Rightarrow \quad x + y + 2 = 0$

Hindi.

(i) सरल रेखा निकाय के समीकरण से $(2x + 3y – 1) + \lambda (3x – 4y – 6) = 0$

यह बिन्दु (3, 2) से गुजरती है। $(6 + 6 – 1) + \lambda (9 – 8 – 6) = 0 \Rightarrow \lambda = \frac{11}{5}$

$(2x + 3y – 1) + \frac{11}{5} (3x – 4y – 6) = 0 \quad \Rightarrow \quad 43x – 29y = 71$

(ii) सरल रेखा निकाय की समीकरण से $(x + 2y + 3) + \lambda (3x + 4y + 7) = 0$

यह $x – y + 8 = 0$ के लम्बवत् है।

अतः $m_1m_2 = – 1 \quad \Rightarrow \quad -\frac{(3\lambda + 1)}{(4\lambda + 2)} (1) = – 1$

$\Rightarrow \quad 3\lambda + 1 = 4\lambda + 2$

$\Rightarrow \quad \lambda = – 1 \quad \Rightarrow \quad (x + 2y + 3) – (3x + 4y + 7) = 0$

$\Rightarrow \quad –2x – 2y – 4 = 0 \Rightarrow \quad x + y + 2 = 0$

E-7. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines $ax + by + c = 0$ and $\ell x + my + n = 0$.

उस त्रिभुज के परिकेन्द्र का बिन्दुपथ ज्ञात कीजिए जिसकी दो भुजाएं निर्देशांक अक्षों के अनुदिश है तथा तीसरी भुजा रेखाओं $ax + by + c = 0$ और $\ell x + my + n = 0$ के प्रतिच्छेद बिन्दु से गुजरती है।

Ans. $2xy(ma-b\ell) + x(an – \ell c) + y (mc – bn) = 0$.

Sol. Let the equation of the third line be $(ax + by + c) + \lambda (\ell x + my + n) = 0$ where λ is a parameter.

it meets the x-axis at A where $y = 0$ and $x = \frac{-(\lambda n + c)}{\lambda \ell + a}$

The triangle OAB is a right angled triangle. Its circumcentre is the midpoint of the hypotenuse. Let it be (α, β)

Hence the locus of is

$2xy(ma-b\ell) + x(an – \ell c) + y (mc – bn) = 0$. **Ans.**

$\Rightarrow 2\alpha = -\left(\frac{\lambda n + c}{\lambda \ell + a}\right)$ and $2\beta = -\left(\frac{\lambda n + c}{\lambda m + b}\right) \Rightarrow \lambda = -\frac{2\alpha a + c}{2\alpha \ell + n} = -\frac{2\beta b + c}{2\beta m + n}$

Hence the locus of (α, β) is $\frac{c + 2ax}{n + 2\ell x} = \frac{c + 2by}{n + 2my}$

$\Rightarrow \quad 2xy(ma-b\ell) + x(an – \ell c) + y (mc – bn) = 0$. **Ans.**

Hindi मानाकि तीसरी रेखा का समीकरण $(ax + by + c) + \lambda (\ell x + my + n) = 0$ जहाँ λ प्राचल है।

यह x-अक्ष को A पर मिलता है जहाँ $y = 0$ और $x = \frac{-(\lambda n + c)}{\lambda \ell + a}$

त्रिभुज OAB एक समकोण त्रिभुज है। इसका परिकेन्द्र कर्ण का मध्य बिन्दु है। माना कि यह (α, β)है।

$\Rightarrow 2\alpha = -\left(\frac{\lambda n + c}{\lambda \ell + a}\right)$ and $2\beta = -\left(\frac{\lambda n + c}{\lambda m + b}\right) \Rightarrow \lambda = -\frac{2\alpha a + c}{2\alpha \ell + n} = -\frac{2\beta b + c}{2\beta m + n}$

अतः (α, β) का बिन्दुपथ $\frac{c + 2ax}{n + 2\ell x} = \frac{c + 2by}{n + 2my}$

$\Rightarrow$ $2xy(ma - b\ell) + x(an - \ell c) + y(mc - bn) = 0$. **Ans.**

Section (F) : Pair of straight lines, Homogenization
खण्ड (F) : सरल रेखाओं के अर्द्धक, समघातीकरण

F-1. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the n^{th} power of the other, then prove that $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0$

यदि रेखाओं $ax^2 + 2hxy + by^2 = 0$ में से एक का ढाल दूसरी के ढाल की n वीं घात है तो सिद्ध कीजिए कि $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0$

Sol. $ax^2 + 2hxy + by^2 = (y - m_1 x)(y - m_2 x)$ given that $m_2 = m_1^n$

Hence $m_1 + m_2 = -\frac{2h}{b} \Rightarrow m_1 + m_1^n = -\frac{2h}{b}$; $m_1 . m_1^n = \frac{a}{b} \Rightarrow m_1 = \left(\frac{a}{b}\right)^{\frac{1}{1+n}}$

Eliminate m_1 from both. we get $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0$

Hindi $ax^2 + 2hxy + by^2 = (y - m_1 x)(y - m_2 x)$ दिया है कि $m_2 = m_1^n$

अतः $m_1 + m_2 = -\frac{2h}{b} \Rightarrow m_1 + m_1^n = -\frac{2h}{b}$; $m_1 . m_1^n = \frac{a}{b} \Rightarrow m_1 = \left(\frac{a}{b}\right)^{\frac{1}{1+n}}$

दोनों से m_1 का विलोप करने पर हम $(ab^n)^{\frac{1}{n+1}} + (a^n b)^{\frac{1}{n+1}} + 2h = 0$ प्राप्त करते है।

F-2. For what value of λ does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represent a pair of straight lines ? Find their
(i) equations,
(ii) point of intersection,
(iii) acute angle between them.

λ के किस मान के लिये समीकरण $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ एक सरल रेखा युग्म को प्रदर्शित करती है। उन सरल रेखाओं के
(i) समीकरण,
(ii) प्रतिच्छेदन बिन्दु,
(iii) उनके मध्य न्यून कोण
भी ज्ञात करो।

Ans. $\lambda = 2$,
(i) $3x - y + 2 = 0$, $4x - 2y + 1 = 0$,
(ii) point of intersection प्रतिच्छेदन बिन्दु $\left(-\frac{3}{2}, -\frac{5}{2}\right)$,
(iii) $\tan^{-1}\left(\frac{1}{7}\right)$,

Sol. $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$
This represents pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ we get $\lambda = 2$
Now $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = (6x - 2y + p)(2x - y + q)$
compair both sides $2p + 6q = 11 \Rightarrow -p - 2q = -5$

solving both we get p = 4, q = $\frac{1}{2}$

Hence required lines are $6x - 2y + 4 = 0$

$\Rightarrow 3x - y + 2 = 0 \Rightarrow 2x - y + \frac{1}{2} = 0 \Rightarrow 4x - 2y + 1 = 0$

solving both equations we get point of intersection $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

Now angle between both lines $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3-2}{1+3\times 2}\right| = \frac{1}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{7}\right)$

Hindi. $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$

यह सरल रेखा युग्म को निरूपित करता है यदि $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \Rightarrow \lambda = 2$

अब $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = (6x - 2y + p)(2x - y + q)$

दोनों तरफ तुलना करने पर $2p + 6q = 11 \Rightarrow -p - 2q = -5$ दोनों को हल करने पर p = 4, q = $\frac{1}{2}$

अतः अभीष्ट रेखाएँ $6x - 2y + 4 = 0 \Rightarrow 3x - y + 2 = 0 \Rightarrow 2x - y + \frac{1}{2} = 0 \Rightarrow 4x - 2y + 1 = 0$

दोनों समीकरणों को हल करने पर प्रतिच्छेद बिन्दु $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

अब दोनों रेखाओं के मध्य कोण $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3-2}{1+3\times 2}\right| = \frac{1}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{7}\right)$

F-3. (i) Find the integral values of 'h' for which $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ represents two real straight lines.

h के पूर्णांक मान ज्ञात कीजिए जबकि $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ दो वास्तविक सरल रेखाओं को प्रदर्शित करता है।

(ii) If the pair of lines represented by equation $k(k-3)x^2 + 16xy + (k+1)y^2 = 0$ are perpendicular to each other, then find k.

यदि $k(k-3)x^2 + 16xy + (k+1)y^2 = 0$ द्वारा प्रदर्शित रेखा युग्म, परस्पर एक दुसरे के लम्बवत् है तब k का मान ज्ञात कीजिए।

Ans. (i) h = 1 (ii) k = 1

Sol. (i) $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ represent pair of lines hence $\Delta = abc + 2fgh - af^2 - by^2 - ch^2 = 0$

$= (h)(4h)(-2) + 2(1)\left(\frac{1}{2}\right)\left(\frac{-5}{2}\right) - h(1)^2 - 4h\left(\frac{1}{2}\right)^2 - (-2)\left(\frac{-5}{2}\right)^2 = 0$

$= -8h^2 - \frac{5}{2} - h - h + \frac{25}{2} = 0 = -8h^2 - 2h + 10 = 0 = 4h^2 + h - 5 = 0 \Rightarrow h = \frac{-5}{4}, 1$

Hence only one integeral value.

(ii) $ax^2 + 2hxy + by^2 = 0$ represents perpendicular straight lines

if $a + b = 0$ i.e. $k(k-3) + (k+1) = 0 \Rightarrow k^2 - 2k + 1 = 0 \Rightarrow k = 1$

Hindi

(i) $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ सरल रेखा युग्म को प्रदर्शित करता है

अतः $\Delta = abc + 2fgh - af^2 - by^2 - ch^2 = 0 = (h)(4h)(-2) + 2(1)\left(\frac{1}{2}\right)\left(\frac{-5}{2}\right) - h(1)^2 - 4h\left(\frac{1}{2}\right)^2 - (-2)\left(\frac{-5}{2}\right)^2 = 0$

$= -8h^2 - \frac{5}{2} - h - h + \frac{25}{2} = 0 = -8h^2 - 2h + 10 = 0 = 4h^2 + h - 5 = 0 \Rightarrow h = \frac{-5}{4}, 1$

अतः केवल एक ही पूर्णांक हल होगा।

(ii) $ax^2 + 2hxy + by^2 = 0$ लम्बवत् रेखाओं को प्रदर्शित करता है,

यदि $a + b = 0$ अर्थात् $k(k-3) + (k+1) = 0 \Rightarrow k^2 - 2k + 1 = 0 \Rightarrow k = 1$

F-4. Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.
सरल रेखा $lx + my + n = 0$ और वक्र $y^2 = 4ax$ के प्रतिच्छेद बिन्दुओं को मूल बिन्दु से मिलाने वाली सरल रेखाओं के समीकरण ज्ञात करो तथा लम्बवत् का प्रतिबन्ध भी ज्ञात कीजिए।

Ans : $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

Ans : $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

Sol. $\frac{lx+my}{-n} = 1 \Rightarrow y^2 = 4ax \times 1 \Rightarrow y^2 = 4ax\left(\frac{\ell x+my}{-n}\right) \Rightarrow 4alx^2 + 4amxy + ny^2 = 0$

for perpendicular $4al + n = 0$ लम्बवत् होने के लिए $4al + n = 0$

F-5. Find the condition that the diagonals of the parallelogram formed by the lines $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$, $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.
रेखाओं $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$, $a'x + b'y + c' = 0$ से बने समान्तर चतुर्भुज के विकर्ण के लम्बवत् होने का प्रतिबन्ध ज्ञात कीजिए तथा समान्तर चतुर्भुज के विकर्ण का समीकरण भी ज्ञात कीजिए।

Ans. $a^2 + b^2 = (a')^2 + (b')^2 \Rightarrow (a + a')x + (b + b')y + c + c' = 0 \Rightarrow (a - a')x + (b - b')y = 0$

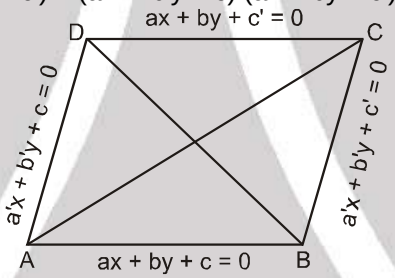
Sol. Equation of diagonals AC is विकर्ण AC का समीकरण है

$(ax + by + c)(a'x + b'y + c) - (a'x + b'y + c')(ax + by + c') = 0$

$x(a + a') + y(b + b') + (c + c') = 0$ .....(1)

Equation of diagonal BD is विकर्ण BD का समीकरण है।

$(ax + by + c)(a'x + b'y + c') - (a'x + b'y + c)(ax + by + c') = 0$



$x(a - a') + y(b - b') = 0$ .....(2)

Now diagonals are perpendicular अब विकर्ण लम्बवत् है

$\frac{(a+a')}{(b+b')} \times \left(\frac{a-a'}{b-b'}\right) = -1 \Rightarrow a^2 + b^2 = (a')^2 + (b')^2.$

PART - II : ONLY ONE OPTION CORRECT TYPE

भाग - II : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

A-1. Mid point of A(0, 0) and B(1024, 2048) is A_1. mid point of A_1 and B is A_2 and so on. Coordinates of A_{10} are.

A(0, 0) और B(1024, 2048) का मध्य बिन्दु A_1 है, A_1 और B का मध्य बिन्दु A_2 है इसी प्रकार A_{10} के निर्देशांक है –

(A) (1022, 2044) (B) (1025, 2050) (C*) (1023, 2046) (D) (1, 2)

Sol. $A_1\left(\frac{0+x_1}{2}, \frac{0+y_1}{2}\right) = \left(\frac{x_1}{2}, \frac{y_1}{2}\right) = \left(1-\frac{1}{2}\right)x_1, \left(1-\frac{1}{2}\right)y_1$

$$A_2\left(\frac{\frac{x_1}{2}+x_1}{2},\frac{\frac{y_1}{2}+y_1}{2}\right)=\left(\frac{3x_1}{2^2},\frac{3y_1}{2^2}\right)=\left(1-\frac{1}{2^2}\right)=\left(\left(1-\frac{1}{2^2}\right)x_1,\left(1-\frac{1}{2^2}\right)y_1\right)$$

$$A_3\left(\frac{\frac{3x_1}{2^2}+x_1}{2},\frac{\frac{3y_1}{2^2}+1}{2}\right)=\left(\frac{7}{2^3}x_1,\frac{7}{2^3}y_1\right)=\left(\left(1-\frac{1}{2^3}\right)x_1,\left(1-\frac{1}{2^3}\right)y_1\right)$$

$$A_{10}\left(\left(1-\frac{1}{2^{10}}\right)x_1,\left(1-\frac{1}{2^{10}}\right)y_1\right)=(1023,2046)$$

A-2. If the points (k, 2 – 2k), (1 – k, 2k) and (–k –4, 6 – 2k) be collinear, the number of possible values of k are

यदि बिन्दु (k, 2 – 2k), (1 – k, 2k) एवं (–k –4, 6 – 2k) संरेखीय हो, तो k के संभव मानों की संख्या है–

(A) 4 (B*) 2 (C) 1 (D) 3

Sol. Since A, B, C are collinear

Slope of AB = Slope of BC A(k, 2 – 2k) B(1 – k, 2k) C(– k– 4, 6 – 2k)

$$\frac{2-2k-2k}{k-1+k}=\frac{2k-6+2k}{1-k+k+4}\Rightarrow\frac{2-4k}{2k-1}=\frac{4k-6}{5}$$

$$\Rightarrow 10-20k=(4k-6)(2k-1)\Rightarrow(4k-6)(2k-1)+10(2k-1)=0\Rightarrow k=\frac{1}{2},-1$$

Hindi. चूँकि A, B, C संरेखीय है

AB की प्रवणता = BC की प्रवणता A(k, 2 – 2k) B(1 – k, 2k) C(– k– 4, 6 – 2k)

$$\frac{2-2k-2k}{k-1+k}=\frac{2k-6+2k}{1-k+k+4}\Rightarrow\frac{2-4k}{2k-1}=\frac{4k-6}{5}$$

$$\Rightarrow 10-20k=(4k-6)(2k-1)\Rightarrow(4k-6)(2k-1)+10(2k-1)=0\Rightarrow k=\frac{1}{2},-1$$

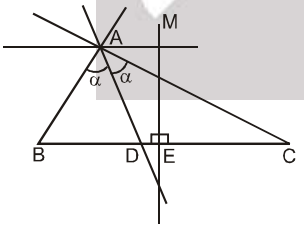
A-3. Given a ΔABC with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals :

(A) 1 (B*) 2 (C) 3 (D) Infinite

दिया गया है एक ΔABC जिसकी भुजायें असमान है। P उस सभी बिन्दुओं का समूह जो B और C से समान दूरी पर है। और Q उन बिन्दुओं का समूह है जो भुजाओं AB और AC से समान दूरी पर है। तब $n(P \cap Q)$ बराबर होगा :

(A) 1 (B*) 2 (C) 3 (D)अनन्त

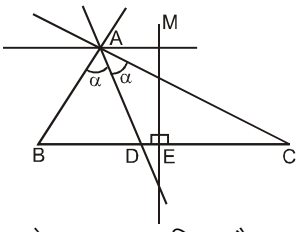
Sol.



P lies on line AD; Q lies on EM

(where E is midpoint of BC) $\Rightarrow$ $n(P \cap Q) = 2$

Hindi.



P, रेखा AD पर स्थित है, Q, EM पर स्थित है।
(जहाँ E , BC का मध्य बिन्दु है) $\Rightarrow$ $n(P \cap Q) = 2$

A-4. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.
एक रेखाखण्ड AB को P और Q क्रमशः समान अनुपात (> 1) में आन्तरिक और बाह्यय विभाजित करता है तथा M, AB का मध्य बिन्दु है।

Statement-1: MP, MB, MQ are in G.P.
कथन-1: MP, MB, MQ गुणोत्तर श्रेणी में है।
Statement-2 AP, AB and AQ are in HP.
कथन-2 AP, AB तथा AQ हरात्मक श्रेणी में है।

(A*) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for
STATEMENT-1Pro
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false
(A*) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण है।
(B) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण नहीं है।
(C) कथन–1 सत्य है, कथन–2 असत्य है।
(D) कथन–1 असत्य है, कथन–2 सत्य है।
(E) सभी कथन असत्य है।

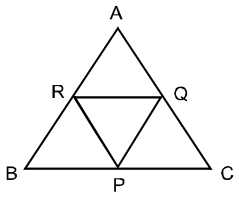
Sol. A — M — P — B — Q
AP, AB, AQ are in H.P.
$AP - \frac{AB}{2}, AB - \frac{AB}{2}, AQ - \frac{AB}{2}$ are in G.P.
MP, MB, MQ are in G.P.

Hindi. A — M — P — B — Q
AP, AB, AQ हरात्मक श्रेणी में है।
$AP - \frac{AB}{2}, AB - \frac{AB}{2}, AQ - \frac{AB}{2}$ गुणोत्तर श्रेणी में है।
MP, MB, MQ गुणोत्तर श्रेणी में है।

A-5. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3)
(A*) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units
एक त्रिभुज के शीर्ष (2, 1), (– 2, 3) एवं (4, – 3) हैं। इस त्रिभुज की भुजाओं के मध्य बिन्दुओं से निर्मित त्रिभुज का क्षेत्रफल है –
(A*) 1.5 वर्ग ईकाई (B) 3 वर्ग ईकाई (C) 6 वर्ग ईकाई (D) 12 वर्ग ईकाई

Sol. Area of the triangle formed by joining the mid points of the sides of the triangle = $\frac{1}{4}$ (area of the triangle)

त्रिभुज की भुजाओं के मध्य बिन्दुओं को मिलाने पर बने त्रिभुज का क्षेत्रफल = $\frac{1}{4}$ (त्रिभुज का क्षेत्रफल)



$= \frac{1}{4} \times \frac{1}{2} \begin{vmatrix} 2 & 1 & 1 \\ -2 & 3 & 1 \\ 4 & -3 & 1 \end{vmatrix} = \frac{1}{8} [2(3+3)-(-2-4)+1(6-12)] = \frac{1}{4} \times 6 = 1.5$ sq.units वर्ग इकाई

Section (B) : Different forms of straight lines and Angle between lines
खण्ड (B) : त्रिभुज और बहुभुज का क्षेत्रफल, सरल रेखाओं के विभिन्न रूप और रेखाओं के मध्य कोण

B-1. A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is :
बिन्दु P (1, 2) से गुजरने वाली एक सरल रेखा का अक्षों के मध्य अंतःखण्ड बिन्दु P पर समद्विभाजित होता है, तो इस रेखा की समीकरण है –
(A) x + 2y = 5 (B) x – y + 1 = 0 (C) x + y – 3 = 0 (D*) 2x + y – 4 = 0

Sol. Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{a}{2} = 1 \Rightarrow a = 2$

$\frac{b}{2} = 2 \Rightarrow b = 4$ Hence $\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow 2x + y - 4 = 0$

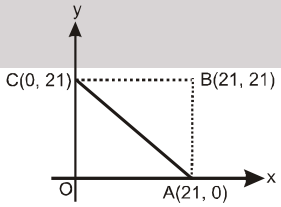
Hindi. माना रेखा का समीकरण $\frac{x}{a} + \frac{y}{b} = 1$ है ; $\frac{a}{2} = 1 \Rightarrow a = 2$

$\frac{b}{2} = 2 \Rightarrow b = 4$ अतः $\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow 2x + y - 4 = 0$

B-2. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is
शीर्ष (0, 0), (0, 21) एवं (21, 0) वाले त्रिभुज के ठीक अन्दर स्थित पूर्णांक बिन्दुओं (अर्थात् ऐसे बिन्दु जिनके x व y दोनों निर्देशांक पूर्णांक हो) की संख्या हैं –
(A) 133 (B*) 190 (C) 233 (D) 105

Sol. The number of integral points that lie in the interior of square OABC is 20 × 20. These points are (x, y) where x, y = 1, 2,...., 20. Out of these 400 points 20 lie on the line AC. Out of the remaining exactly half lie in ΔABC.

$\therefore$ number of integeral point in the triangle OAC = $\frac{1}{2}$ [20 × 20 – 20] = 190

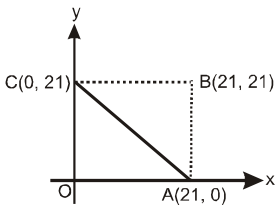


Alternative Solution
There are 19 points that lie in the interior of ΔABC and on the line x + y = 19, 18 point that lie on the line x + y = 18 and so on. Thus, the number of desired points is

$19 + 18 + 17 + ... + 2 + 1 = \frac{20 \times 19}{2} = 190.$

Hindi. वर्ग OABC के अन्दर स्थित पूर्णांक बिन्दुओं की संख्या 20 × 20 है। ये बिन्दु (x, y) है जहाँ x, y = 1, 2, ..., 20 इन 400 बिन्दुओं में से 20, रेखा AC पर स्थित है। शेष में से ठीक आधे ΔABC पर स्थित है।

$\therefore$ ΔOAC में पूर्णांक बिन्दुओं की संख्या है = $\frac{1}{2}[20 \times 20 - 20] = 190$



वैकल्पिक विधि

यहाँ 19 बिन्दु है जो ΔABC के अन्दर है तथा रेखा $x + y = 19$ पर है। 18 बिन्दु $x + y = 18$ पर स्थित है। तथा इसी प्रकार अभीष्ट बिन्दुओं की संख्या $19 + 18 + 17 + ... + 2 + 1 = \frac{20 \times 19}{2} = 190.$

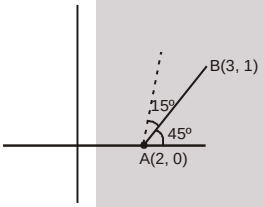
B-3. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15º. The equation of the line in the new position is :

(A) $x - \sqrt{3}y - 2 = 0$ (B) $x - 2y - 2 = 0$ (C*) $\sqrt{3}x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2}x - y - 2\sqrt{2} = 0$

दो बिन्दुओं A (2, 0) तथा B (3, 1) से जाने वाली एक सरल रेखा को A के सापेक्ष वामावर्त दिशा में 15º कोण पर घुमाया जाता है तो नई स्थिति में रेखा की समीकरण होगी –

(A) $x - \sqrt{3}y - 2 = 0$ (B) $x - 2y - 2 = 0$ (C*) $\sqrt{3}x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2}x - y - 2\sqrt{2} = 0$

Sol. Slope of AB is $\tan\theta = \frac{1-0}{3-2} = 1$



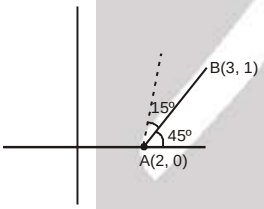
$\theta = 45^\circ$

Hence equation of new line is

$y - 0 = \tan 60^\circ(x - 2)$

$y = \sqrt{3}x - 2\sqrt{3} \Rightarrow \sqrt{3}x - y - 2\sqrt{3} = 0$

Hindi. AB की प्रवणता $\tan\theta = \frac{1-0}{3-2} = 1$



$\theta = 45^\circ$

अतः नयी स्थिति में रेखा का समीकरण

$y - 0 = \tan 60^\circ(x - 2)$

$\Rightarrow$ $y = \sqrt{3}x - 2\sqrt{3} \Rightarrow \sqrt{3}x - y - 2\sqrt{3} = 0$

B-4. In a ΔABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation $x + 2y = 16$. If the mid point of BC is (5, 6), then the equation of BC is
(A) $2x + y = 16$ (B*) $x + y = 11$ (C) $2x - y = 4$ (D) $x + y = 10$
एक त्रिभुज ABC की भुजाओं AB व AC की समीकरणें क्रमशः $2x + 3y = 29$ तथा $x + 2y = 16$ है यदि भुजा BC का मध्य बिन्दु (5, 6) हो तो भुजा BC की समीकरण है–
(A) $2x + y = 16$ (B*) $x + y = 11$ (C) $2x - y = 4$ (D) $x + y = 10$

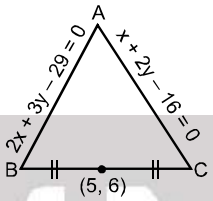
Sol. Let $B(x_1, y_1)$ and $C(x_2, y_2)$

$\therefore$ $2x_1 + 3y_1 - 29 = 0$ .....(i)

and $x_2 + 2y_2 - 16 = 0$ .....(ii)

$\because$ mid-point of BC is (5, 6)

$\therefore$ $x_1 + x_2 = 10$ .....(iii)



and $y_1 + y_2 = 12$ .....(iv)

Put the value of x_2 and y_2 in (ii), we get

$10 - x_1 + 2(12 - y_1) - 16 = 0$

$x_1 + 2y_1 = 18$ .....(v)

Now on solving (i) and (v), we get $x_1 = 4$ and $y_1 = 7$

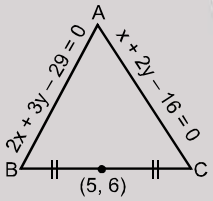
$\therefore$ B(4, 7)

$\therefore$ equation of line BC is $y - 6 = \frac{7-6}{4-5}(x - 5) \Rightarrow x + y = 11$

Hindi. माना $B(x_1, y_1)$ और $C(x_2, y_2)$ $\therefore$ $2x_1 + 3y_1 - 29 = 0$ .....(i)

और $x_2 + 2y_2 - 16 = 0$ .....(ii)

$\because$ BC का मध्यबिन्दु (5, 6) है। $\therefore$ $x_1 + x_2 = 10$ .....(iii)



और $y_1 + y_2 = 12$ .....(iv)

x_2 व y_2 का मान (ii) में रखने पर हम पाते हैं।

$10 - x_1 + 2(1_2 - y_1) - 16 = 0$ $x_1 + 2y_1 = 18$ .....(v)

अब (i) तथा (v) को हल करने पर हम पाते हैं $x_1 = 4$ व $y_1 = 7$

$\therefore$ B(4, 7)

$\therefore$ रेखा BC की समीकरण है $y - 6 = \frac{7-6}{4-5}(x - 5) \Rightarrow x + y = 11$

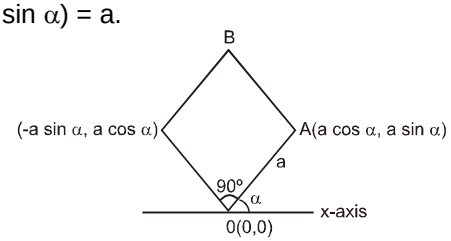
B-5. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is :
यदि 'a' भुजा का वर्ग x-अक्ष के ऊपर स्थित है तथा उसका एक शीर्ष मूलबिन्दु पर है। मूल बिन्दु से गुजरने वाली भुजा x अक्ष की धनात्मक दिशा से $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ कोण बनाती है। इसके मूल बिन्दु से न गुजरने वाले विकर्ण की समीकरण है–

(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ (B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ (D*) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

Sol. Equation of AC AC का समीकरण $\Rightarrow y - a\sin\alpha = \frac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}(x - a\cos\alpha)$

$y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a(\sin\alpha\cos\alpha + \sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha)$

$y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a.$



B-6. Find equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120º with the positive direction of x-axis.

सरल रेखा का समीकरण ज्ञात कीजिए जबकि मूल बिन्दु से लम्ब की लम्बाई 4 इकाई तथा रेखा धनात्मक x-अक्ष से 120º का कोण बनाती है।

(A) $\sqrt{3}x - y = 0$ (B*) $\sqrt{3}x + y = 8$ (C) $x + \sqrt{3}y = 8$ (D) $x - \sqrt{3}y = 8$

Sol. $x\cos\alpha + y\sin\alpha = p$

$x\cos 30^\circ + y\sin 30^\circ = 4$

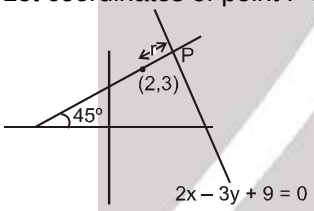
$\sqrt{3}x + y = 8$

B-7. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :

बिन्दु (2, 3) की रेखा $x - y + 1 = 0$ के अनुदिश रेखा $2x - 3y + 9 = 0$ से दूरी है –

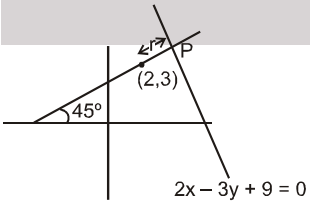
(A) $5\sqrt{3}$ (B*) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

Sol. Let coordinates of point P by parametric $P(2 + r\cos 45^\circ, 3 + r\sin 45^\circ)$



It satisfies the line $2x - 3y + 9 = 0$; $2\left(2 + \frac{r}{\sqrt{2}}\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9 = 0 \Rightarrow r = 4\sqrt{2}$

Hindi. माना P के निर्देशांक प्राचलिक रूप में $P(2 + r\cos 45^\circ, 3 + r\sin 45^\circ)$



यह रेखा $2x - 3y + 9 = 0$ को संतुष्ट करता है। $2\left(2 + \frac{r}{\sqrt{2}}\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9 = 0 \Rightarrow r = 4\sqrt{2}$

B-8. If a point P(x, y) from where line drawn cuts coordinate axes at A and B (with A on x-axis and B on y-axis) satisfies $\alpha.\frac{x^2}{PB^2}+\beta\frac{y^2}{PA^2}=1$, then $\alpha + \beta$ is

यदि बिन्दु P(x, y) से जहाँ रेखा निर्देशांक अक्षों को A तथा B पर काटती है तथा $\alpha.\frac{x^2}{PB^2}+\beta\frac{y^2}{PA^2}=1$, को संतुष्ट करती है तब $\alpha + \beta$ का मान है– (जहाँ A, x-अक्ष पर है तथा B, y-अक्ष पर है)

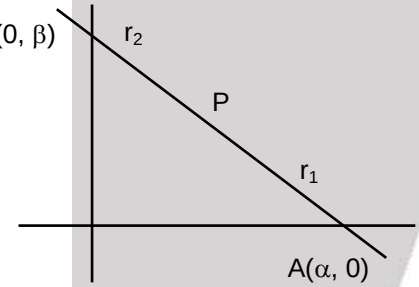
(A) 1 (B*) 2 (C) 3 (D) 4

Sol. P(h, k)

$(h + r\cos\theta, k + r\sin\theta)$

For point A

बिन्दु A से



$k + r_1 h\,\theta = 0 \quad \Rightarrow \quad r_1 = -\frac{k}{\sin\theta} \quad \Rightarrow \quad r_1 = -\frac{k}{\sin\theta}$

For point B. बिन्दु B से $h_1 + r_2\cos\theta = 0$

$r_2 = -\frac{h}{\cos\theta} \quad \Rightarrow \quad \cos^2\theta + \sin^2\theta = 1 \quad \Rightarrow \quad \frac{h^2}{r_2^2}+\frac{k^2}{r_1^2}=1$

$$\frac{x^2}{PB^2}+\frac{y^2}{AP^2}=1$$

B-9. Two particles start from the point (2, – 1), one moving 2 units along the line x + y = 1 and the other 5 units along the line x – 2y = 4. If the particles move towards increasing y, then their new positions are
बिन्दु (2, – 1) से दो कण चलते है। उनमें से एक रेखा x + y = 1 के अनुदिश 2 इकाई दुरी चलता है तथा अन्य रेखा x – 2y = 4 के अनुदिश 5 इकाई दूरी चलता है। यदि दोनो कण y के बढने की दिशा में हो तब इनकी नयी स्थिति है–

(A*) $\left(2-\sqrt{2},\sqrt{2}-1\right),\ \left(2\sqrt{5}+2,\sqrt{5}-1\right)$ (B) $\left(2\sqrt{5}+2,\sqrt{5}-1\right),\ \left(2\sqrt{2},\sqrt{2}+1\right)$

(C) $\left(2+\sqrt{2},\ \sqrt{2}+1\right),\ \left(2\sqrt{5}+2,\sqrt{5}+1\right)$ (D) none of these इनमें से कोई नहीं।

Sol. P(2, –1) goes 2 units along x + y = 1 upto A and 5 units along x – 2y = 4 upto B.
slope of PA = – 1 = tan 135°

slope of PB = 1/2 = tan θ $\sin\theta=\frac{1}{\sqrt{5}},\cos\theta=\frac{2}{\sqrt{5}}$

The coordinates of B i.e. $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ हैं $\left(2\sqrt{5}+2,\sqrt{5}-1\right)$

The coordinates of A i.e. $(x_1 + r\cos135^0, y_1 + r\sin135^0)$ are $(2-\sqrt{2}, \sqrt{2}-1)$

Hindi. P(2, –1), 2 इकाई x + y = 1 के अनुदिश A तक जाता है तथा 5 इकाई x – 2y = 4 के अनुदिश B तक जाता है

PA की प्रवणता = – 1 = tan 135⁰

PB की प्रवणता = 1/2 = tan θ $\qquad \sin\theta = \frac{1}{\sqrt{5}}, \cos\theta = \frac{2}{\sqrt{5}}$

B के निर्देशांक $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ हैं $\left(2\sqrt{5}+2, \sqrt{5}-1\right)$

A के निर्देशांक $(x_1 + r\cos135^0, y_1 + r\sin135^0)$ हैं $(2-\sqrt{2}, \sqrt{2}-1)$

B-10. Equation of a straight line passing through the origin and making with x – axis an acute angle twice the size of the angle made by the line y = (0.2) x with the x – axis, is :

रेखा y = (0.2) x द्वारा x –अक्ष के साथ बनाए गए कोण का दुगना कोण बनाने वाली एवं मूलबिन्दु से गुजरने वाली सरल रेखा का समीकरण है :

(A) y = (0.4) x (B*) y = (5/12) x (C) 6y – 5x = 0 (D) 6y + 5x = 0

Sol. here यहाँ $\tan\theta = \frac{1}{5}$ $\quad \therefore \quad \tan 2\theta = \frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} = \frac{5}{12}$

$\therefore$ required line अतः अभीष्ठ रेखा $y = \frac{5x}{12}$

B-11. The points A (1, 3) and C (5,1) are the oppositive vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is

बिन्दुओं A (1, 3) तथा C (5,1) आयत के विपरीत शीर्ष है। अन्य दो शीर्षो और प्रवणता 2 से गुजरने वाली रेखा का समीकरण है–

(A) 2x+y–8 = 0 (B*) 2x–y–4 = 0 (C) 2x–y+4 = 0 (D) 2x+y = 0

Sol. Mid point ≡ (3, 2). Equation is 2x–y–4 = 0.

मध्य बिन्दु ≡ (3, 2) है समीकरण 2x–y–4 = 0 है।

B-12. The point (–4,5) is the vertex of a square and one of its digonals is 7x–y+8 = 0. The equation of the other diagonal is

बिन्दु P(–4,5) वर्ग का एक शीर्ष है तथा इसका विकर्ण 7x–y+8 = 0. है तो इसके अन्य विकर्ण का समीकरण है –

(A) 7x–y+23 = 0 (B) 7y + x =30 (C*) 7y + x =31 (D) x –7y =30

Sol. Point (–4, 5) does not lie on the diagonal 7x–y+8= 0, so point will lie on the other diagonal also diagonal is perpendicular

$\therefore$ Slope of other diagonal = –1/7

$\therefore$ Equation of other diagonal is

$y-5 = -\frac{1}{7}(x+4) \Rightarrow 7y+x=31.$

Hindi बिन्दु (–4, 5) विकर्ण पर स्थित नहीं 7x–y+8= 0 इसलिए बिन्दु, अन्य विकर्ण पर स्थित होगा तथा विकर्ण लम्बवत् है –

$\therefore$ अन्य विकर्ण की प्रवणता = –1/7

$\therefore$ अन्य विकर्ण का समीकरण

$y - 5 = -\frac{1}{7}(x+4) \Rightarrow 7y+x=31.$

B-13. Two straight lines x + 2y = 2 and x + 2y = 6 are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally
दो रेखाएं x + 2y = 2 और x + 2y = 6 दी गई है तब रेखा का समीकरण जो दी गई रेखाओं के समान्तर है और उन रेखाओं के मध्य की दुरी को 2 : 1 में अन्तः विभाजित करती है।

(A) 3x + 6y + 8 = 0 (B*) 3x + 6y = 14
(C) 3x + 6y + 14 = 0 (D) 3x + 2y = 10

Sol. Let $L_1 \equiv x + 2y - 2 = 0$ and $L_2 \equiv x + 2y - 6 = 0$ and parallel line $x + 2y + k = 0$

$\therefore \quad \left|\frac{k+1}{\sqrt{5}}\right| = 2\left|\frac{k+6}{\sqrt{5}}\right| \quad \Rightarrow \quad k + 2 = 2k + 12 \text{ or } k + 2 = -2k - 12$

$k = -10 \text{ or } k = -\frac{14}{3} \quad \Rightarrow \quad x + 2y = 10 \text{ or } 3x + 6y - 14 = 0.$

Hindi. माना $L_1 \equiv x + 2y - 2 = 0$ और $L_2 \equiv x + 2y - 6 = 0$ तथा समान्तर रेखा $x + 2y + k = 0$ है

$\therefore \quad \left|\frac{k+1}{\sqrt{5}}\right| = 2\left|\frac{k+6}{\sqrt{5}}\right| \quad \Rightarrow \quad k + 2 = 2k + 12$ या $k + 2 = -2k - 12$

$k = -10$ या $k = -\frac{14}{3} \quad \Rightarrow \quad x + 2y = 10$ या $3x + 6y - 14 = 0.$

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot
खण्ड (C) : बिन्दु की स्थिति, रैखिक असमिकाऐं लम्बवत् दूरी, प्रतिबिम्ब और लम्बपाद

C-1. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0 \ \forall \ a \in R, b > 0$ are :
'b' के उन मानों का समुच्चय, जिनके लिये मूलबिन्दु तथा बिन्दु (1, 1) सरल रेखा $a^2x + a\,by + 1 = 0$ के एक ही ओर स्थित हैं (सभी $a \in R, b > 0$ के लिये) :

(A) $b \in (2, 4)$ (B*) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

Sol. $a^2x + a\,by + 1 = 0$ origin and (1, 1) lies on same side. $a^2 + ab + 1 > 0 \quad \forall\ a \in R$
$D < 0 \Rightarrow b^2 - 4 < 0 \Rightarrow b \in (-2, 2)$ but $b > 0 \Rightarrow b \in (0, 2)$

Hindi. $a^2x + a\,by + 1 = 0$ मूल बिन्दु और (1, 1) एक ही ओर स्थित है। $a^2 + ab + 1 > 0 \ \forall\ a \in R$
$D < 0 \Rightarrow b^2 - 4 < 0 \Rightarrow b \in (-2, 2)$ परन्तु $b > 0 \Rightarrow b \in (0, 2)$

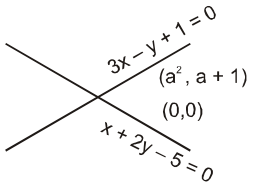
C-2. The point $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin, then

(A) $a \geq 1$ or $a \leq -3$ (B*) $a \in (-3, 0) \cup (1/3, 1)$
(C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$

बिन्दु $(a^2, a + 1)$ रेखाओं $3x - y + 1 = 0$ एवं $x + 2y - 5 = 0$ के मध्य उस कोण में स्थित होगा जिसमें मूल बिन्दु स्थित है, तो

(A) $a \geq 1$ या $a \leq -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$
(C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$

Sol. Origin & $(a^2, a + 1)$ lies same side w.r.t. to given lines
$a^2 + 2a + 2 - 5 < 0 \Rightarrow a^2 + 2a - 3 < 0$
$\Rightarrow (a + 3)(a - 1) < 0$
$\Rightarrow a \in (-3, 1)$
$3a^2 - (a + 1) + 1 > 0 \Rightarrow 3a^2 - a > 0$



$\Rightarrow$ $a(3a - 1) > 0$

$\Rightarrow$ $a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

take intersection we get $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

Hindi. मूल बिन्दु एवं $(a^2, a + 1)$ दी गई रेखा के सापेक्ष एक ही ओर स्थित होगें।

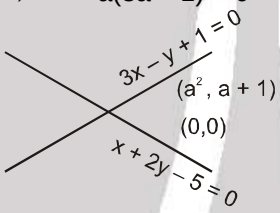
$a^2 + 2a + 2 - 5 < 0$ $\Rightarrow$ $a^2 + 2a - 3 < 0$

$\Rightarrow$ $(a + 3)(a - 1) < 0$

$\Rightarrow$ $a \in (-3, 1)$

$3a^2 - (a + 1) + 1 > 0$ $\Rightarrow$ $3a^2 - a > 0$

$\Rightarrow$ $a(3a - 1) > 0$



$\Rightarrow$ $a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

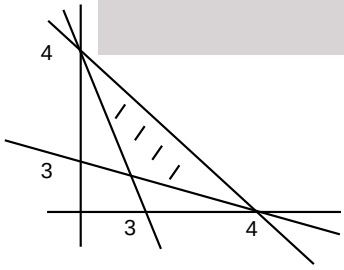
सर्वनिष्ठ लेने पर $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

C-3. Find area of region represented by $3x + 4y > 12$, $4x + 3y > 12$ and $x + y < 4$

$3x + 4y > 12$, $4x + 3y > 12$ और $x + y < 4$ से निरूपित क्षेत्र का क्षेत्रफल है –

(A*) $\frac{8}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$

Sol.



$3x + 4y = 12$

$4x + 3y = 12$

$x - y = 0$

$(x, y) = \left(\frac{12}{7}, \frac{12}{7}\right)$

Area = $\frac{1}{2}.1.4 - \frac{1}{2}.1.\frac{12}{7} = 2 - \frac{6}{7} = \frac{8}{7}$

C-4. The image of the point A (1, 2) by the line mirror y = x is the point B and the image of B by the line mirror y = 0 is the point (α, β), then :
(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C*) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$
रेखा दर्पण y = x में बिन्दु A (1, 2) का प्रतिबिम्ब B है तथा रेखा दर्पण y = 0 में B के प्रतिबिम्ब के निर्देशांक (α, β) है, तो–
(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$

Sol. Image of A(1, 2) in line mirror y = x is (2, 1) ; Image of B(2, 1) in y = 0 (x – axes) is 2, – 1)
Hence, $\alpha = 2, \beta = -1$

Hindi. रेखा y = x में बिन्दु A(1, 2) का प्रतिबिम्ब (2, 1) होगा ; B(2, 1) का रेखा y = 0 अर्थात् (x – अक्ष) में प्रतिबिम्ब (2, – 1) होगा।
अतः, $\alpha = 2, \beta = -1$

C-5. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are x – y + 5 = 0 and x + 2y = 0 respectively. If the point A is (1, –2), then the equation of the line BC is :
ΔABC की भुजाओं AB व AC के लम्ब समद्विभाजकों की समीकरणें क्रमशः x – y + 5 = 0 तथा x + 2y = 0 है यदि बिन्दु A (1, –2) हो, तो भुजा BC की समीकरण होगी –
(A*) 14x + 23y = 40 (B) 14x – 23y = 40 (C) 23x + 14y = 40 (D) 23x – 14y = 40

Sol. Image of A in x – y + 5 = 0 is $\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{2}\right) = -8 \Rightarrow x = -7, y = 6$

Image of A(1, – 2) in x + 2y = 0

$$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5} \Rightarrow x = \frac{11}{5}, y = \frac{2}{5}$$

Hence equation of BC is $y - 6 = \frac{(6 - 2/5)}{(-7 - 11/5)}(x + 7)$

$$y - 6 = \frac{28}{-46}(x + 7) \Rightarrow y - 6 = \frac{-14}{23}(x + 7) \Rightarrow \Rightarrow 14x + 23y - 40 = 0$$

Hindi. x – y + 5 = 0 में बिन्दु A का प्रतिबिम्ब $\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{2}\right) = -8 \Rightarrow x = -7, y = 6$

x + 2y = 0 में A(1, – 2) का प्रतिबिम्ब

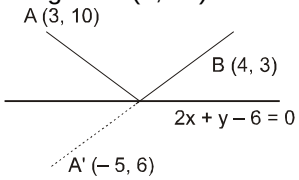
$$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5} \Rightarrow x = \frac{11}{5}, y = \frac{2}{5}$$

अतः BC का समीकरण $y - 6 = \frac{(6 - 2/5)}{(-7 - 11/5)}(x + 7)$

$$y - 6 = \frac{28}{-46}(x + 7) \Rightarrow y - 6 = \frac{-14}{23}(x + 7) \Rightarrow \Rightarrow 14x + 23y - 40 = 0$$

C-6. A light beam emanating from the point A(3, 10) reflects from the straight line 2x + y – 6 = 0 and then passes through the point B(4, 3). The equation of the reflected beam is x + 3y – λ = 0, then the value of λ is
बिन्दु A(3, 10) से उत्सर्जित होने वाली एक प्रकाश किरण, सरल रेखा 2x + y – 6 = 0 से परावर्तित होकर बिन्दु B(4, 3) से गुजरती है, तो परावर्तित प्रकाश किरण का समीकरण x + 3y – λ = 0 हो तो λ का मान है–
(A) 11 (B) 12 (C*) 13 (D) 14

Sol. Image of A(3, 10) in $2x + y - 6 = 0$



$$\frac{x-3}{2} = \frac{y-10}{1} = -2\left(\frac{6+10-6}{2^2+1^2}\right)$$

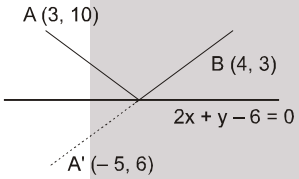
$$\frac{x-3}{2} = \frac{y-10}{1} = -4$$

A' = (– 5, 6)

Equation of A'B is $y - 3 = \left(\frac{6-3}{-5-4}\right)(x-4) \Rightarrow y - 3 = -(x-4)$

$3y - 9 = -x + 4 \Rightarrow x + 3y - 13 = 0$

Hindi. A(3, 10) का रेखा $2x + y - 6 = 0$ में प्रतिबिम्ब



$$\frac{x-3}{2} = \frac{y-10}{1} = -2\left(\frac{6+10-6}{2^2+1^2}\right)$$ से

$$\frac{x-3}{2} = \frac{y-10}{1} = -4$$

A' = (– 5, 6)

A'B का समीकरण $y - 3 = \left(\frac{6-3}{-5-4}\right)(x-4) \Rightarrow y - 3 = -\frac{1}{3}(x-4)$

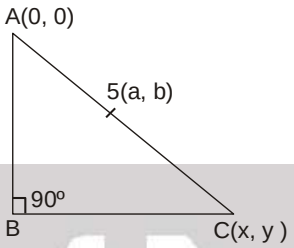
$3y - 9 = -x + 4 \Rightarrow x + 3y - 13 = 0$

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus
खण्ड (D) : केन्द्रक लम्बकेन्द्र, परिकेन्द्र बहिष्केन्द्र, बिन्दुपथ

D-1. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin, then the co–ordinates of C are :
यदि त्रिभुज ABC का लम्बकेंद्र 'B' तथा परिकेंद्र 'S' (a, b) हो तथा A मूल बिन्दू हो, तो C के निर्देशांक होंगे :

(A*) (2a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2}, 0\right)$ (D) none इनमें से कोई नहीं

Sol. By given information. Since in ΔABC, B is other centre. Hence $\angle B = 90^o$
Circum centre is S(a, b)

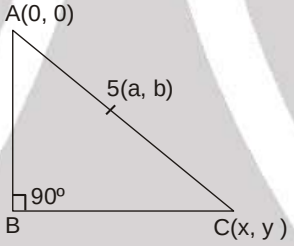


$\frac{x+0}{2} = a \Rightarrow x = 2a$

$\frac{y+0}{2} = b \Rightarrow y = 2b$

Hence, $c(x, y) \equiv (2a, 2b)$

Hindi. दी गई जानकारी के अनुसार चूंकि ΔABC में, B, लम्बकेन्द्र है। अतः $\angle B = 90^o$
परिकेन्द्र S(a, b)



$\frac{x+0}{2} = a \Rightarrow x = 2a$

$\frac{y+0}{2} = b \Rightarrow y = 2b$

अतः, $c(x, y) \equiv (2a, 2b)$

D-2. A triangle ABC with vertices A (– 1, 0),B (– 2, 3/4) & C (– 3, – 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :
(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D*) none of these
एक त्रिभुज ABC जिसके शीर्ष A (– 1, 0),B (– 2, 3/4) व C (– 3, – 7/6) है, का लम्बकेन्द्र H है तो ΔBCH का लम्बकेन्द्र होगा।
(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D*) इनमें से कोई नहीं

Sol. If H is orthocentre of triangle ABC, then orthocentre of triangle BCH is point A
यदि ΔABC का लम्ब केन्द्र H हो तो ΔBCH का लम्ब केन्द्र बिन्दु A होगा।

D-3. Find locus of centroid of ΔABC, if B(1, 1), C(4, 2) and A lies on the line y = x + 3.
ΔABC के केन्द्रक का बिन्दुपथ ज्ञात कीजिए यदि B(1, 1), C(4, 2) और A, रेखा y = x + 3 पर स्थित है।
(A) 3x + 3y + 1 = 0 (B) x + y = 3 (C*) 3x – 3y + 1 = 0 (D) x – y = 3

Sol. Let centroid is G(h, k) and $A(x_1, y_1)$ on line y = x + 3.

$\therefore \quad h = \frac{1+4+x_1}{3}, k = \frac{1+2+y_1}{3} \quad \therefore \quad x_1 = 3h - 5, y_1 = 3k - 3$

Now, $A(x_1, y_1)$ lies on $y = x + 3 \Rightarrow 3k - 3 = 3h - 5 + 3 \Rightarrow 3x - 3y + 1 = 0$.

Hindi माना केन्द्रक G(h, k) तथा $A(x_1, y_1)$ रेखा $y = x + 3$ पर स्थित है

$\therefore \quad h = \frac{1+4+x_1}{3}, k = \frac{1+2+y_1}{3} \quad \therefore \quad x_1 = 3h - 5, y_1 = 3k - 3$

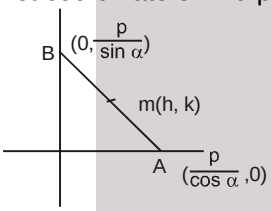
अब, $A(x_1, y_1)$ रेखा पर स्थित है $y = x + 3 \Rightarrow 3k - 3 = 3h - 5 + 3 \Rightarrow 3x - 3y + 1 = 0$.

D-4. The locus of the mid-point of the distance between the axes of the variable line $x \cos\alpha + y \sin\alpha = p$, where p is constant, is

एक चर रेखा $x \cos\alpha + y \sin\alpha = p$, (जहाँ p अचर है) के अक्षों के मध्य अंतःखण्ड के मध्य बिन्दु का बिन्दुपथ है–

(A) $x^2 + y^2 = 4p^2$ (B*) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{2}{p^2}$

Sol. Let coordinate of mid point is m(h, k)



$2h = \frac{p}{\cos\alpha} \Rightarrow \cos\alpha = \frac{p}{2h}$

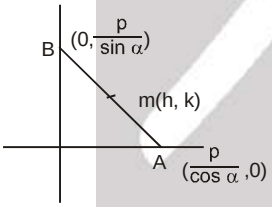
$2k = \frac{p}{\sin\alpha} \Rightarrow \sin\alpha = \frac{p}{2k}$

Squareing and add.

$\frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$

Locus of p(h, k) $\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

Hindi. माना मध्य बिन्दु m(h, k) है



$2h = \frac{p}{\cos\alpha} \Rightarrow \cos\alpha = \frac{p}{2h}$

$2k = \frac{p}{\sin\alpha} \Rightarrow \sin\alpha = \frac{p}{2k}$

वर्ग करके जोड़ने पर

$\frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$

p(h, k) का बिन्दुपथ $\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

D-5. Find the locus of a point which moves so that sum of the squares of its distance from the axes is equal to 3.

बिन्दु का बिन्दुपथ ज्ञात कीजिए जबकि अक्षो से इसके वर्गो का योगफल 3 के बराबर है–

(A) $x^2 + y^2 = 9$ (B*) $x^2 + y^2 = 3$ (C) $|x|+|y|=3$ (D) $x^2 - y^2 = 3$

Sol. Let point (h,k)

Distance from y-axis = |h|

---------- x ----- = (k)

$\therefore h^2+k^2 = 3$

$x^2 + y^2 = 3$

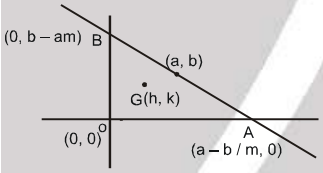
D-6. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is :
एक स्थिर बिन्दु (a, b) से गुजरने वाली एक चर सरल रेखा निर्देशी अक्षों को क्रमशः A और B पर काटती है, यदि 'O' मूलबिन्दु हो, तो त्रिभुज OAB के केंद्रक का बिन्दुपथ है–

(A*) $bx + ay - 3xy = 0$ (B) $bx + ay - 2xy = 0$

(C) $ax + by - 3xy = 0$ (D) $ax + by - 2xy = 0$

Sol. equation of line AB ; $y - b = m(x - a)$

$\therefore \quad G\left(\frac{a-\frac{b}{m}}{3}, \frac{b-am}{3}\right) \Rightarrow h = \frac{a-\frac{b}{m}}{3},\ k = \frac{b-am}{3}$

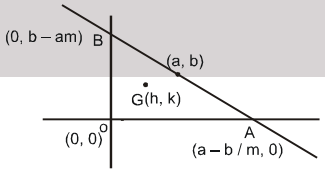


on eleminating 'm' we get required locus

$bh + ak - 3hk = 0 \quad \Rightarrow \quad bx + ay - 3xy = 0$

Hindi. AB का समीकरण $y - b = m(x - a)$

$\therefore \quad G\left(\frac{a-\frac{b}{m}}{3}, \frac{b-am}{3}\right) \Rightarrow h = \frac{a-\frac{b}{m}}{3},\ k = \frac{b-am}{3}$



m का विलोपन करने पर अभीष्ठ बिन्दु का बिन्दुपथ प्राप्त होगा

$bh + ak - 3hk = 0 \quad \Rightarrow \quad bx + ay - 3xy = 0$

D-7. Consider a triangle ABC, whose vertices are A(–2, 1), B(1, 3) and C(x, y). If C is a moving point such that area of ΔABC is constant, then locus of C is :
(A*) Straight line (B) Circle (C) Ray (D) Parabola
माना कि एक त्रिभुज ABC है। जिसके शीर्ष A(–2, 1), B(1, 3) और C(x, y) यदि एक बिन्दु C इस तरह से गति करता है त्रिभुज ΔABC का क्षेत्रफल नियत रहे तो बिन्दु C का बिन्दुपथ होगा :
(A) सरल रेखा (B) वृत्त (C) किरण (D) परवलय

Sol. The vertex 'C' of triangle should move on a line parallel to the base AB
त्रिभुज का शीर्ष 'C', आधार AB के समान्तर रेखा पर गति करता हैi.e. अर्थात् $3y = 2x + \lambda$

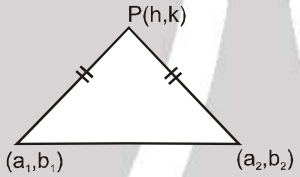
D-8. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)\, x + (b_1 - b_2)\, y + c = 0$, then the value of 'c' is :
यदि बिन्दुओं (a_1, b_1) तथा (a_2, b_2) से समान दूरी पर स्थित बिन्दु का बिन्दुपथ $(a_1 - a_2)\, x + (b_1 - b_2)\, y + c = 0$ है, तब 'c' का मान है –

(A*) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Sol. $(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$2h(a_1 - a_2) + 2k(b_1 - b_2) + \left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right) = 0$



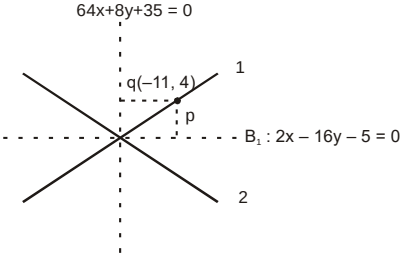
compare with $(a_1 - a_2)x + (b_1 - b_2)\, y + c = 0$

$$c = \frac{\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)}{2}.$$

Section (E) : Angle Bisector, condition of concurrency, family of straight lines
खण्ड (E) : कोण अर्द्धक, संगामी होने का प्रतिबन्ध, सरल रेखाओं का निकाय

E-1. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through $(-11, 4)$, the equation of acute angle bisector of L_1 & L_2 is :
(A*) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$ (C) $2x + 16y + 5 = 0$ (D) $2x + 16y - 5 = 0$
दो रेखाओं L_1 व L_2 के कोण समद्विभाजक के समीकरण $2x - 16y - 5 = 0$ तथा $64x + 8y + 35 = 0$ है। यदि रेखा L_1 बिंदु $(-11, 4)$ से गुजरती है तो L_1 व L_2 के न्यूनकोण समद्विभाजक का समीकरण है –
(A*) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$
(C) $2x + 16y + 5 = 0$ (D) $2x + 16y - 5 = 0$

Sol. $p = \left|\frac{-22 - 64 - 5}{\sqrt{2^2 + (-16)^2}}\right| = \frac{91}{\sqrt{260}}$



$q = \left|\frac{-64\times11+8\times4+35}{\sqrt{64^2+8^2}}\right| = \frac{637}{\sqrt{4160}} = \frac{159.25}{\sqrt{260}}$

p < q Hence 2x – 16y – 5 = - is acute angle besectory

p < q अतः 2x – 16y – 5 = - न्यून कोण अर्द्धक है।

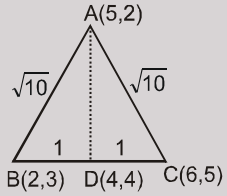
E-2. The equation of the internal bisector of ∠BAC of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is

(A) 2x + y + 12 = 0 (B) x + 2y – 12 = 0 (C*) 2x + y – 12 = 0 (D) 2x – y – 12 = 0

शीर्षों A(5, 2), B(2, 3) व C(6, 5) से बने त्रिभुज ABC के कोण ∠BAC के आंतरिक कोण समद्विभाजक की समीकरण है–

(A) 2x + y + 12 = 0 (B) x + 2y – 12 = 0 (C*) 2x + y – 12 = 0 (D) 2x – y – 12 = 0

Sol. $\frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{1}$ ∴ D(4, 4)



Equation of AD : $y - 4 = \frac{2}{-1}(x-4)$

AD का समीकरण $y - 4 = \frac{2}{-1}(x-4) \Rightarrow$ $y - 4 = -2x + 8 \Rightarrow$ $2x + y = 12$

E-3. The equation of the bisector of the angle between two lines 3 x – 4 y + 12 = 0 and 12 x – 5 y + 7 = 0 which contains the point (– 1, 4) is :

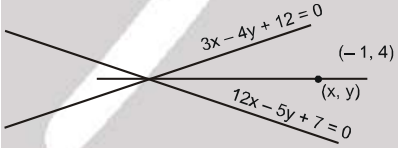
दो रेखाओं 3 x – 4 y + 12 = 0 एवं 12 x – 5 y + 7 = 0 के बीच बिन्दु (– 1, 4) को समाहित करने वाले कोण के अर्धक का समीकरण है –

(A*) 21x + 27y – 121 = 0 (B) 21x – 27y + 121 = 0

(C) 21x + 27y + 191 = 0 (D) $\frac{-3x+4y-12}{5} = \frac{12x-5y+7}{13}$

Sol. at (– 1, 4) पर

3x – 4y + 12 < 0and तथा 12x – 5y + 7 < 0



$\Rightarrow \frac{3x-4y+12}{12x-5y+12} > 0$ at (– 1, 4) पर

So we have to take the bisector with + sign अतः धनात्मक चिन्ह लेकर कोण अर्धक ज्ञात करना होगा

$\frac{3x-4y+12}{5} = \frac{12x-5y+7}{13} \Rightarrow$ 21x + 27y – 121 = 0

E-4. The least positive value of t so that the lines x = t + a, y + 16 = 0 and y =ax (where a is real variable) are concurrent is

t का न्यूनतम धनात्मक मान ज्ञात कीजिए जबकि रेखाएं x = t + a, y + 16 = 0 और y =ax (जहाँ a वास्तविक चर है) संगामी है–

(A) 2 (B) 4 (C) 16 (D*) 8

Sol. (d) $t = -\left(\frac{a^2+16}{a}\right) = -\left(a+\frac{16}{a}\right)$

Least positive value, न्यूनतम धनात्मक मान = 8

E-5. Consider the family of lines $5x + 3y - 2 + \lambda_1 (3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -
माना कि रेखाओं का निकाय $5x + 3y - 2 + \lambda_1 (3x - y - 4) = 0$ और $x - y + 1 + \lambda_2(2x - y - 2) = 0$ सरल रेखा का समीकरण, दोनों निकायों में है
(A) $25x - 62y + 86 = 0$ (B) $62x - 25y + 86 = 0$
(C) $25x - 62y = 86$ (D*) $5x - 2y - 7 = 0$

Sol. Lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ are concurrent at $(1, -1)$ and lines $x - y + 1 + \lambda_2(2x - y - 2) = 0$ are concurrent at $(3, 4)$. Thus equation of line common to both family is,

$y - 4 = \frac{-1-4}{1-3}(x - 3)$ i.e., $5x - 2y - 7 = 0$.

Hindi. रेखा $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ बिन्दु $(1, -1)$ पर संगामी है तथा रेखाऐं $x - y + 1 + \lambda_2(2x - y - 2) = 0$ बिन्दु $(3, 4)$ पर संगामी है तबा दोनो निकाय में उभयनिष्ठ रेखा का समीकरण है।

$y - 4 = \frac{-1-4}{1-3}(x - 3)$ i.e., $5x - 2y - 7 = 0$.

E-6. The equation of a line of the system $2x + y + 4 + \lambda (x - 2y - 3) = 0$ which is at a distance $\sqrt{10}$ units from point A(2, –3) is
(A) $3x + y + 1 = 0$ (B*) $3x - y + 1 = 0$ (C) $y - 3x + 1 = 0$ (D) $y + 3x - 2 = 0$
निकाय $2x + y + 4 + \lambda (x - 2y - 3) = 0$ की उस रेखा की समीकरण जो बिन्दु A(2, –3) से $\sqrt{10}$ इकाई दूरी पर स्थित है।
(A) $3x + y + 1 = 0$ (B*) $3x - y + 1 = 0$ (C) $y - 3x + 1 = 0$ (D) $y + 3x - 2 = 0$

Sol. $(2 + \lambda) x + (1 - 2\lambda) y + (4 - 3\lambda) = 0$

distance from point A is $= \left|\frac{(2+\lambda)\times 2 - 3\times(1-2\lambda) + (4-3\lambda)}{\sqrt{(2+\lambda)^2 + (1-2\lambda)^2}}\right| = \sqrt{10} \Rightarrow \lambda = 1$

Hence, the required line is $3x - y + 1 = 0$

Hindi. $(2 + \lambda) + (1 - 2\lambda) y + (4 - 3\lambda) = 0$

बिन्दु A से दूरी है $\left|\frac{(2+\lambda)\times 2 - 3\times(1-2\lambda) + (4-3\lambda)}{\sqrt{(2+\lambda)^2 + (1-2\lambda)^2}}\right| = \sqrt{10} \Rightarrow \lambda = 1$

अतः अभीष्ट रेखा की समीकरण $3x - y + 1 = 0$

E-7. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, are concurrent at the point
यदि $3a + 2b + 4c = 0$ हो, तो $ax + by + c = 0$ से प्रदर्शित सरल रेखाएँ निम्न में से किस बिन्दु पर संगामी हैं –
(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D*) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Sol. $ax + by + c = 0 \Rightarrow \frac{3a}{4} + \frac{b}{2} + c = 0$

Compare both $(x, y) \equiv \left(\frac{3}{4}, \frac{1}{2}\right)$. Hence given family passes through $\left(\frac{3}{4}, \frac{1}{2}\right)$.

Hindi. $ax + by + c = 0 \Rightarrow \frac{3a}{4} + \frac{b}{2} + c = 0$

दोनों की तुलना करने पर $(x, y) \equiv \left(\frac{3}{4}, \frac{1}{2}\right)$. अतः दिया गया निकाय बिन्दु $\left(\frac{3}{4}, \frac{1}{2}\right)$ से गुजरेगा।

E-8. Find the equation of a straight line which passes through the point of intersection of the straight lines $x + y - 5 = 0$ and $x - y + 3 = 0$ and perpendicular to the straight line intersecting x-axis at the point $(-2, 0)$ and the y-axis at the point $(0, -3)$,
एक रेखा जोकि रेखाओं $x + y - 5 = 0$ तथा $x - y + 3 = 0$ के प्रतिच्छेदी बिन्दु से गुजरती है तथा उस रेखा के लम्बवत् है जो कि x-अक्ष को बिन्दु $(-2, 0)$ व y-अक्ष को $(0, -3)$ पर काटती है, का समीकरण है

(A) $2x + 3y + 10 = 0$ (B*) $2x - 3y + 10 = 0$

(C) $2x - 5y + 10 = 0$ (D) $2x + 5y + 10 = 0$

Sol. By family of lines requred line is : $(x + y - 5) + \lambda(x - y + 3) = 0$

equation of other line is $\frac{x}{-2} + \frac{y}{-3} = 1 \Rightarrow 3x + 2y + 6 = 0$

Both are $\perp$ hence $m_1m_2 = -1 \quad \Rightarrow \quad -\left(\frac{\lambda+1}{1-\lambda}\right)\left(\frac{-3}{2}\right) = -1 \Rightarrow 3\lambda + 3 = -2 + 2\lambda \Rightarrow \lambda = -5$

Hence requred line $-4x = 6y - 20 = 0 \quad \Rightarrow \quad 2x - 3y + 10 = 0$

Hindi. सरल रेखा निकाय होगा $(x + y - 5) + \lambda(x - y + 3) = 0$ अन्य रेखा का समीकरण $\frac{x}{-2} + \frac{y}{-3} \Rightarrow 3x + 2y + 6 = 0$

दोनों लम्बवत् है अतः $m_1m_2 = -1 \quad \Rightarrow \quad -\left(\frac{\lambda+1}{1-\lambda}\right)\left(\frac{-3}{2}\right) = -1 \Rightarrow 3\lambda + 3 = -2 + 2\lambda \Rightarrow \lambda = -5$

अतः अभीष्ट रेखा $-4x = 6y - 20 = 0 \quad \Rightarrow \quad 2x - 3y + 10 = 0$

E-9. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(A) Above the x-axis at a distance of 3/2 from it

(B) Above the x-axis at a distance of 2/3 from it

(C*) Below the x-axis at a distance of 3/2 from it

(D) Below the x-axis at a distance of 2/3 from it

रेखा x-अक्ष के समान्तर तथा रेखाओं $ax + 2by + 3b = 0$ और $bx - 2ay - 3a = 0$ के प्रतिच्छेद बिन्दु से गुजरती है, जहाँ $(a, b) \neq (0, 0)$

(A) x-अक्ष के ऊपर इससे 3/2 इकाई दुरी पर (B) x-अक्ष से ऊपर इससे 2/3 दुरी पर

(C*) x-अक्ष से नीचे इससे 3/2 दुरी पर (D) x-अक्ष के नीचे इससे 2/3 दुरी पर

Sol. The lines passing throug the intersection of the lines $ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

रेखाओं $ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$ के प्रतिच्छेद बिन्दु से गुजरने वाली रेखा का समीकरण

$\Rightarrow \quad (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

$$y\left(2b + \frac{2a^2}{b}\right) + 3b\frac{3a^2}{b} = 0$$

$$y\left(\frac{2b^2 + 2a^2}{b}\right) + 3b\frac{3a^2}{b} = 0, \quad \left(\frac{2b^2 + 2a^2}{b}\right) = \left(\frac{3b^2 + 3a^2}{b}\right)$$

$$y = \frac{-3\left(a^2 + b^2\right)}{2\left(b^2 + a^2\right)} = \frac{-3}{2}, \; y = -\frac{3}{2}$$

Section (F) : Pair of straight lines, Homogenization
खण्ड (F) : सरल रेखाओं के अर्द्धक, समघातीकरण

F-1. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =
यदि रेखा युग्म $ax^2 + 10xy + y^2 = 0$ की एक रेखा की ढाल, दूसरी की ढाल की चार गुना है तो a =
(A) 1 (B) 2 (C) 4 (D*) 16

Sol. $m_1 + m_2 = -10 \quad \Rightarrow \quad m_1m_2 = \frac{a}{1}$

given $m_1 = 4m_2 \quad \Rightarrow \quad m_2 = -2, m_1 = -8, a = 16$

Hindi. $m_1 + m_2 = -10 \quad \Rightarrow \quad m_1m_2 = \frac{a}{1}$

दिया गया है $m_1 = 4m_2 \quad \Rightarrow \quad m_2 = -2, m_1 = -8, \ a = 16$

F-2. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is
समीकरण $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ सरल रेखाओं के युग्म को प्रदर्शित करती है तब उनके मध्य दुरी है–

(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C*) 2 (D) $2\sqrt{3}$

Sol. $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0 \quad \Rightarrow \quad (x + \sqrt{2}\,y + p)(x + \sqrt{2}\,y + q) = 0$

$p + q = 4 \quad \Rightarrow \quad pq = 1$

Distance between parallel lines is $\left|\frac{p-q}{\sqrt{3}}\right| = \frac{\sqrt{(p+q)^2 - 4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$

Hindi. $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0 \quad \Rightarrow \quad (x + \sqrt{2}\,y + p)(x + \sqrt{2}\,y + q) = 0$

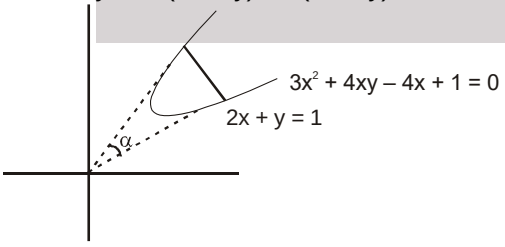
$p + q = 4 \quad \Rightarrow \quad pq = 1$

समान्तर रेखाओं के मध्य दूरी $\left|\frac{p-q}{\sqrt{3}}\right| = \frac{\sqrt{(p+q)^2 - 4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$

F-3. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :
सरल रेखा $2x + y = 1$ तथा वक्र $3x^2 + 4xy - 4x + 1 = 0$ के प्रतिच्छेदन बिंदुओं को मूलबिंदु से जोड़ने पर प्राप्त सरल रेखाओं के मध्य कोण है –

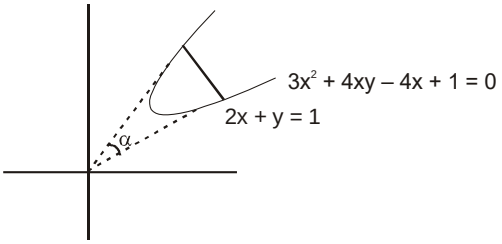
(A*) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Sol. Homogenize given curve with given line
$3x^2 + 4xy - 4x(2x + y) + 1(2x + y)^2 = 0$



$3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy = 0$
$-x^2 + 4xy + y^2 =$
coeff. x^2 + coeff. $y^2 = 0$
Hence angle is 90º

Hindi. वक्र को दी गई रेखा के साथ समघात बनाने पर
$3x^2 + 4xy - 4x(2x + y) + 1(2x + y)^2 = 0$



$3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy = 0$
$-x^2 + 4xy + y^2 =$
x^2 का गुणांक + y^2 का गुणांक = 0
अतः कोण 90^o है।

PART - III : MATCH THE COLUMN

भाग - III : कॉलम को सुमेलित कीजिए (MATCH THE COLUMN)

1.

	Column –I		**Column – II**
(A)	The points A(–4, –1), B (–2, –4), C (4, 0) and D(2, 3) are the vertices of	(p)	Square
(B)	The figure formed by the lines 2x + 5y + 4 = 0, 5x + 2y + 7 = 0, 2x + 5y + 3 = 0 and 5x + 2y + 6 = 0 is	(q)	Rectangle
(C)	A quadrilateral whose diagonal are perpendicular bisectors of each other must be	(r)	Rhombus
(D)	A quadrilateral whose diagonals are angle bisector of sides and bisect each other must be	(s)	Parallelogram
	स्तम्भ–I		**स्तम्भ– II**
(A)	बिन्दु A(–4, –1), B (–2, –4), C (4, 0) एवं D(2, 3) किसके शीर्ष है –	(p)	वर्ग
(B)	रेखाओं 2x + 5y + 4 = 0, 5x + 2y + 7 = 0, 2x + 5y + 3 = 0 तथा 5x + 2y + 6 = 0 से बनने वाली आकृति हैं–	(q)	आयत
(C)	चतुर्भुज जिसके विकर्ण लम्बसमद्विभाजक होते हैं	(r)	समचतुर्भुज
(D)	चतुर्भुज जिसके विकर्ण भुजाओं के कोण अर्द्धक हैं और एक दुसरे को समद्विभाजित करते है–	(s)	समान्तर चतुर्भुज

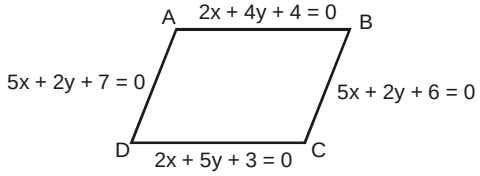
Ans. **(A) (q, s) (B) (r, s) (C) (r, s) (D) (r, s)**

Sol. **(A)** AB = $\sqrt{4+9} = \sqrt{13}$; BC = $\sqrt{36+16} = 2\sqrt{13}$

CD = $\sqrt{4+9} = \sqrt{13}$; AD = $\sqrt{36+16} = 2\sqrt{13}$

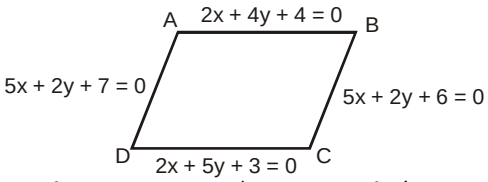
AC = $\sqrt{64+1} = \sqrt{65}$; BD = $\sqrt{16+49} = \sqrt{65}$ its rectangle आयत है।

(B) Distance between lines AB & CD = Distance between lines AD and BC



$\Rightarrow$ ABCD is a rhombus, also side AD is not perpendicular to DC hence not a square.

Hindi : रेखाओं AB व CD के मध्य दूरी = रेखाओं AD तथा BC के मध्य दूरी



$\Rightarrow$ ABCD एक समचतुर्भुज है, साथ ही भुजा AD, DC के लम्बवत् नहीं है अतः वर्ग नहीं है।

(C) **Obviously** स्पष्टतयाः

(D) **Obviously** स्पष्टतयाः

2. **Match The Followings :**

	Column –I		**Column – II**	
(A)	P lies on the line $y = x$ and Q lies on $y = 2x$. The equation for the locus of the mid point of PQ, if $\|PQ\| = 4$, is $25x^2 - \lambda xy + 13y^2 = 4$, then λ equals	(p)	36
(B)	The line $(K + 1)^2 x + ky - 2K^2 - 2 = 0$ passes through a point (α, β) regardless of value K. Then $(\alpha - \beta)$ equals :	(q)	6
(C)	Let two lines be $C_1 : 3x - 4y + 1 = 0$ and $C_2 : 8x + 6y + 1 = 0$ suppose (α, β) is a point which is at unit distance from each of the given lines. then sum of all possible value of α is	(r)	–4/5
(D)	If distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is $25\sqrt{2}$, then 'a/5' is equal to	(s)	– 1

Ans. (A) $\rightarrow$ (p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

स्तम्भ मिलान कीजिए –

	स्तम्भ –I		**स्तम्भ – II**	
(A)	बिन्दु P रेखा y = x पर स्थित है और बिन्दु Q रेखा y = 2x पर स्थित है। तो PQ के मध्य बिन्दु के बिन्दुपथ का समीकरण होगा, यदि |PQ| = 4, है $25x^2 - \lambda xy + 13y^2 = 4$, तब λ बराबर	(p)	36
(B)	(α, β) से गुजरने पर रेखा $(K+1)^2 x + ky - 2K^2 - 2 = 0$, K से स्वतन्त्र है तब $(\alpha - \beta)$ बराबर होगा :	(q)	6
(C)	यदि दो रेखाये $C_1 : 3x - 4y + 1 = 0$ तथा $C_2 : 8x + 6y + 1 = 0$ माना (α, β) कोई बिन्दु जो कि प्रत्येक रेखा से ईकाई दूरी पर स्थित है तब α के सभी संभावित मानो का योगफल, होगा	(r)	–4/5
(D)	यदि समांतर सरल रेखाओं $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ के मध्य दूरी $25\sqrt{2}$ है, तो a/5 बराबर है –	(s)	– 1

Ans. (A) $\rightarrow$ (p), (B) $\rightarrow$ (q), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

Sol. (A) $P = (h, h)$; $Q \equiv (k, 2k)$ $\Rightarrow$ $(PQ)^2 = 16 = (h-k)^2 + (h-2k)^2$

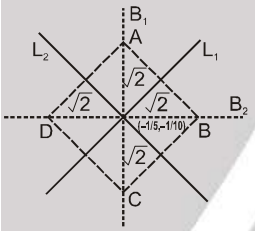
Mid point of $PQ \equiv M \equiv \left(\frac{h+k}{2}, \frac{h+2k}{2}\right) = (x, y)$

$h + k = 2x$; $h + 2k = 2y$ $\Rightarrow$ $k = 2(y - x)$; $h = 4x - 2y$ $\Rightarrow$ $25x^2 + 13y^2 - 36xy = 4$

(B) $(k^2 + 2k + 1)h + ky - 2k^2 - 2 = 0 \Rightarrow k^2(x - 2) + k(2x + y) + x - 2 = 0 \Rightarrow x = 2$ and $y = -4$

(C) $B_1 = 2x + 14y - 1 = 0$; $B_2 = 14x - 2y + 3 = 0$

$\tan\theta_1 = \frac{-1}{7}$ and $\tan\theta_2 = 7$



sum of all possible value of α are $= 0 + \frac{6}{5} - \frac{8}{5} - \frac{2}{5} = \frac{-4}{5}$.

(D) Lines represented by given equation are $x + y + a = 0$ and $x + y - 9a = 0$

$\therefore$ distance between these parallel lines is $= \left|\frac{10a}{\sqrt{2}}\right| = 25\sqrt{2}$

Hindi : Sol.

(A) $P = (h, h)$; $Q \equiv (k, 2k)$ $\Rightarrow$ $(PQ)^2 = 16 = (h-k)^2 + (h-2k)^2$

Mid point of $PQ \equiv M \equiv \left(\frac{h+k}{2}, \frac{h+2k}{2}\right) = (x, y)$

$h + k = 2x$; $h + 2k = 2y$ $\Rightarrow$ $k = 2(y - x)$; $h = 4x - 2y$ $\Rightarrow$ $25x^2 + 13y^2 - 36xy = 4$

(B) $K = 0$ $\Rightarrow$ $x = 2$

$K = -1$ $\Rightarrow$ $-y - 2 - 2 = 0$ $\Rightarrow$ $y = -4$

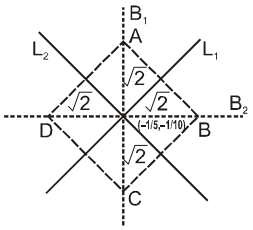
बिन्दु = (2, –4)

जाँच : $2k^2 + 2 + 4K - 4K - 2K^2 - 2 = 0$

$\alpha - \beta = 6$

(C) $B_1 = 2x + 14y - 1 = 0$; $B_2 = 14x - 2y + 3 = 0$

$\tan\theta_1 = \frac{-1}{7}$ और $\tan\theta_2 = 7$



α के सभी संभावित मानों का योगफल $= 0 + \frac{6}{5} - \frac{8}{5} - \frac{2}{5} = \frac{-4}{5}$.

(D) समीकरण द्वारा प्रदर्शित रेखायें $x + y + a = 0$ तथा $x + y - 9a = 0$ है,

अतः इन समान्तर रेखाओं के मध्य की दूरी $= \left|\frac{10a}{\sqrt{2}}\right| = 25\sqrt{2}$ है।

Exercise-2

Marked questions are recommended for Revision.
चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

PART - I : ONLY ONE OPTION CORRECT TYPE

भाग-I : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

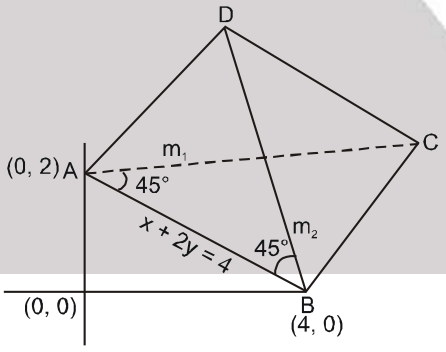
1. On the portion of the straight line $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co–ordinates :

सरल रेखा $x + 2y = 4$ द्वारा अक्षों के मध्य काटे गये भाग पर एक वर्ग, मूल बिन्दु के विपरीत ओर, बनाया जाता है, तो इसके विकर्णों के प्रतिच्छेद बिन्दु के निर्देशांक होगें :

(A) (2, 3) (B) (3, 2) (C*) (3, 3) (D) (2,2)

Sol. $\tan 45° = \left|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right| \Rightarrow \pm 1 = \frac{2m+1}{2-m} \Rightarrow m = \frac{1}{3}, -3$

$\therefore$ Equation of AC AC का समीकरण



$y - 2 = \frac{1}{3}(x) \Rightarrow x - 3y + 6 = 0$ .... (i)

Equation of BD BD का समीकरण

$y = -3(x - 4) \Rightarrow 3x + y - 12 = 0$ .... (ii)

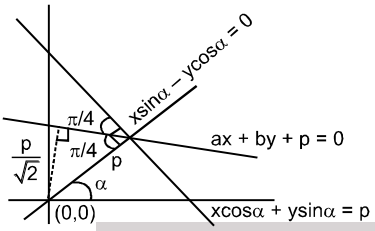
(i) & व (ii) से $x = 3$ & $y = 3$

2. If the straight line ax + by + p = 0 and xcosα + ysinα = p enclosed an angle of $\frac{\pi}{4}$ and the line xsinα – ycosα = 0 meets them at the same point, then $a^2 + b^2$ is

यदि सरल रेखा ax + by + p = 0 तथा xcosα + ysinα = p के मध्य कोण $\frac{\pi}{4}$ है तथा सरल रेखा xsinα – ycosα = 0 उन रेखाओं को एक ही बिन्दु पर मिलती है, तब $a^2 + b^2$ है–

(A) 4 (B) 3 (C*) 2 (D) 1

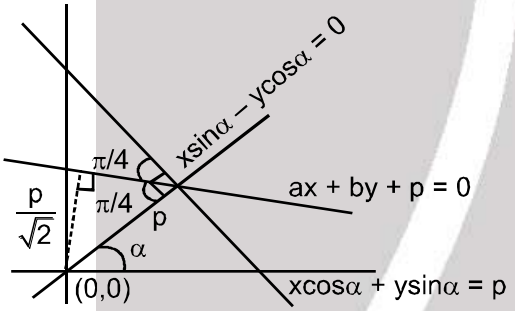
Sol.



So from diagram

Perpendicular distance from origin to line ax + by + p = 0 is $\frac{p}{\sqrt{2}}$, then $\left|\frac{p}{\sqrt{a^2+b^2}}\right| = \left|\frac{p}{\sqrt{2}}\right| \Rightarrow a^2 + b^2 = 2$

Hindi



लेखा चित्र से

मूल बिन्दु से सरल रेखा ax + by + p = 0 पर डाले गये लम्ब की लम्बाई $\frac{p}{\sqrt{2}}$, तब $\left|\frac{p}{\sqrt{a^2+b^2}}\right| = \left|\frac{p}{\sqrt{2}}\right| \Rightarrow a^2 + b^2 = 2$

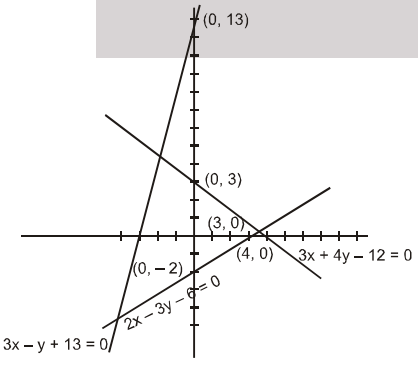
3. A ΔABC is formed by the lines 2x – 3y – 6 = 0 , 3x – y + 3 = 0 and 3x + 4y – 12 = 0. If the points P(α, 0) and Q(0, β) always lie on or inside the ΔABC, then ;

एक त्रिभुज ABC, रेखाओं 2x – 3y – 6 = 0 , 3x – y + 3 = 0 व 3x + 4y – 12 = 0 द्वारा बनाया जाता है, यदि बिंदु P(α, 0) तथा Q(0, β) सदैव ΔABC पर या उसके अंदर स्थित है, तो

(A) α ∈ [–1, 2] & β∈ [–2, 3] (B) α ∈ [–1, 3] & β∈ [–2, 4]

(C) α ∈ [–2, 4] & β∈ [–3, 4] (D*) α ∈ [–1, 3] & β∈ [–2, 3]

Sol. $L_1 = 2x – 3y – 6 = 0$



$L_2 = 3x – y + 3 = 0$ ⇒ $L_3 = 3x + 4y – 12 = 0$

Hence अतः α ∈[– 1, 3] ⇒ β ∈[– 2, 3]

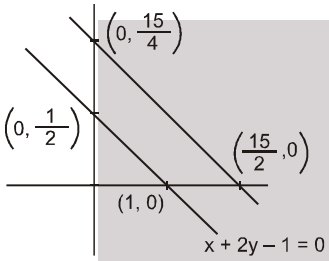
4. If $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines x + 2y = 1 and 2x + 4y = 15 is

(A*) $-\frac{4\sqrt{2}}{3}<t<\frac{5\sqrt{2}}{6}$ (B) $0<t<\frac{5\sqrt{2}}{6}$ (C) $-\frac{4\sqrt{2}}{5}<t<0$ (D) $-\frac{4\sqrt{2}}{3}<t<\frac{\sqrt{2}}{6}$

यदि बिन्दु $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ एक रेखा पर स्थित कोई बिन्दु हो, तो t के उन मानों का समुच्चय जिनके लिये P, समांतर रेखाओं x + 2y = 1 तथा 2x + 4y = 15 के बीच स्थित है –

(A) $-\frac{4\sqrt{2}}{3}<t<\frac{5\sqrt{2}}{6}$ (B) $0<t<\frac{5\sqrt{2}}{6}$ (C) $-\frac{4\sqrt{2}}{5}<t<0$ (D) $-\frac{4\sqrt{2}}{3}<t<\frac{\sqrt{2}}{6}$

Sol. Point $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ lies between given line



Hence $L_1(P) = \left(1+\frac{t}{\sqrt{2}}\right)+2\left(2+\frac{t}{\sqrt{2}}\right)-1=0$

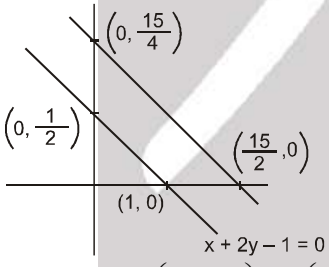
$5+\frac{3t}{\sqrt{2}}-1=0\Rightarrow t=-\frac{4\sqrt{2}}{3}$

Now, $L_2(P) = 2\left(1+\frac{t}{\sqrt{2}}\right)+4\left(2+\frac{t}{\sqrt{2}}\right)-15=0 \Rightarrow \quad 10+\frac{6t}{\sqrt{2}}-15=0\Rightarrow t=\frac{5\sqrt{2}}{6}$

and $L_1(P) \times L_2(P) < 0$

Hence $t \in$.

Hindi. बिन्दु $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ दी गई रेखाओं के मध्य स्थित है



अतः $L_1(P) = \left(1+\frac{t}{\sqrt{2}}\right)+2\left(2+\frac{t}{\sqrt{2}}\right)-1=0$

$5+\frac{3t}{\sqrt{2}}-1=0\Rightarrow t=-\frac{4\sqrt{2}}{3}$

अब, $L_2(P) = 2\left(1+\frac{t}{\sqrt{2}}\right)+4\left(2+\frac{t}{\sqrt{2}}\right)-15=0\Rightarrow 10+\frac{6t}{\sqrt{2}}-15=0\Rightarrow t=\frac{5\sqrt{2}}{6}$

and $L_1(P) \times L_2(P) < 0$ अतः $t \in \left(\frac{-4\sqrt{2}}{3},\frac{5\sqrt{2}}{6}\right)$.

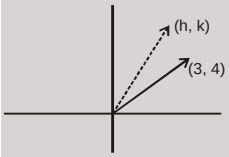
5. The point A(4, 1) undergoes following transformations successively :
(i) reflection about line y = x
(ii) translation through a distance of 3 units in the positive direction of x-axis
(iii) rotation through an angle 105° in anti-clockwise direction about origin O.
Then the final position of point A is

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D*) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

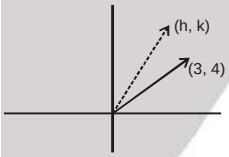
बिन्दु A(4, 1) क्रमागत रूप से निम्न रूपान्तरणों से गुजरता है
(i) रेखा y = x के सापेक्ष प्रतिबिम्ब होता है।
(ii) x-अक्ष की धनात्मक दिशा में 3 इकाई विस्थापित होता है।
(iii) मूल बिन्दु O के सापेक्ष वामावर्त दिशा में 105° कोण से घूर्णित होता है।
तब बिन्दु A के अन्तिम स्थिति में निर्देशांक है

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D*) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

Sol. (i) After reflection about line y = x position of point will be (1, 4)
(ii) After this step (3, 4)
(iii) $h = 4\sqrt{2}\cos 150°$, $k = 4\sqrt{2}\sin 150°$
$h = -2\sqrt{6}$, $k = 2\sqrt{2}$



Hindi. (i) रेखा y = x के सापेक्ष प्रतिबिंब लेने पर बिन्दु की स्थिति (1, 4) है।
(ii) इस चरण के बाद (3, 4)
(iii) $h = 4\sqrt{2}\cos 150°$, $k = 4\sqrt{2}\sin 150°$
$h = -2\sqrt{6}$, $k = 2\sqrt{2}$

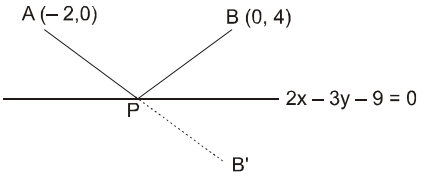


6. Given two points A ≡ (–2, 0) and B ≡ (0, 4), then find coordinate of a point P lying on the line 2x – 3y = 9 so that perimeter of Δ APB is least.
दो बिन्दु A ≡ (–2, 0) तथा B ≡ (0, 4) दिये गये हैं, सरल रेखा 2x – 3y = 9 पर स्थित वह बिन्दु P जिसके लिये त्रिभुज APB का परिमाप न्यूनतम है, के निर्देशांक होंगे –

(A) $\left(\frac{42}{13}, -\frac{11}{3}\right)$ (B) $\left(\frac{84}{13}, -\frac{74}{13}\right)$ (C*) $\left(\frac{21}{17}, -\frac{37}{17}\right)$ (D) (0, – 3)

Sol. Both A & B are same side of line 2x – 3y – 9 = 0
Now, permeter of ΔAPB will be least when pts A, P, B will be collinear. Let B' is image of B

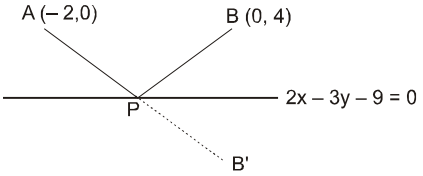
Then $\frac{x-0}{2} = \frac{y-4}{-3} = -2\left(\frac{0-12-9}{2^2+(-3)^2}\right) \Rightarrow B'\left(\frac{84}{13}, \frac{-74}{13}\right)$



Now equation of AB' is $y = \frac{-74}{110}(x + 2)$

point of intersection of given line & Q is $P\left(\frac{21}{17},\frac{-37}{17}\right)$.

Hindi. A तथा B दोनों रेखा $2x - 3y - 9 = 0$ एक ही ओर है।
अब ΔAPB का परिमाप न्यूनतम होगा, जब A , P, B संरेख है।
माना B का प्रतिबिम्ब B′ है



तब $= \frac{x-0}{2} = \frac{y-4}{-3} - 2\left(\frac{0-12-9}{2^2+(-3)^2}\right) \Rightarrow B'\left(\frac{84}{13},\frac{-74}{13}\right)$

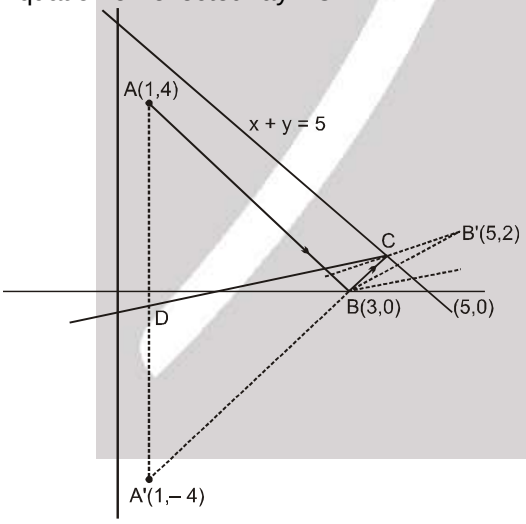
AB' का समीकरण = $\frac{-74}{110}(x+2)$

दी गई रेखा और Q का प्रतिच्छेद बिन्दु $P\left(\frac{21}{17},\frac{-37}{17}\right)$.

7. A ray of light is sent from the point (1, 4). Upon reaching the x-axis, the ray is reflected from the point (3, 0). This reflected ray is again reflected by the line $x + y = 5$ and intersect y-axis at P. Find the co-ordinate of P.
बिन्दु (1, 4) से एक रेखा किरण जाती है तथा x-अक्ष पर पहुंचकर यह बिन्दु (3, 0) से परावर्तित होती है पुनः यह रेखा $x + y = 5$ से परावर्तित होती है जो y-अक्ष को बिन्दु P पर प्रतिच्छेद करती है तो P के निर्देशांक ज्ञात कीजिए।

(A) $\left(\frac{1}{2},0\right)$ (B*) $\left(0,\frac{-1}{2}\right)$ (C) $\left(0,\frac{1}{3}\right)$ (D) $\left(2,\frac{1}{-2}\right)$

Sol. Equation of reflected ray BC



$y - 0 = \frac{0+4}{3-1}(x-3)$

$y = 2x - 6$ .....(1)

Point 'C' is intersection
Point of (1) and $x + y = 5$

$\therefore \quad C\left(\frac{11}{3},\frac{4}{3}\right)$

Now, Image of B(3, 0) w.r.t. $x + y = 5$.

$$\frac{x-3}{1} = \frac{y-0}{1} = -2\left(\frac{3+0-5}{1+1}\right)$$

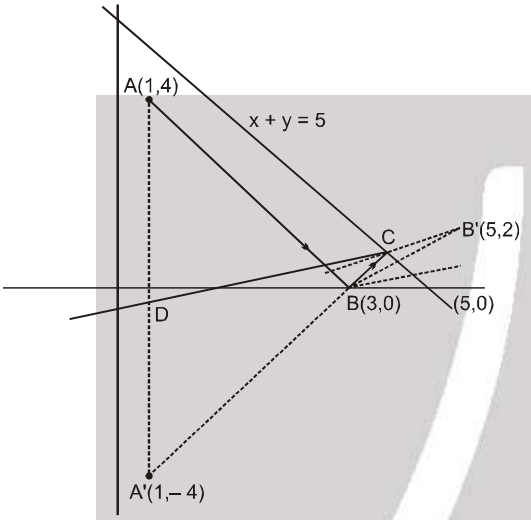
$x = 5, y = 2$

$\therefore$ B' (5, 2)

Now, equation of reflected line CD is $y - 2 = \frac{\frac{4}{3}-2}{\frac{11}{3}-5}(x-5) \Rightarrow 2(y-2) = (x-5) \Rightarrow x - 2y = 1$

$\therefore$ $D\left(0, \frac{-1}{2}\right)$.

Hindi. परावर्तित किरण BC का समीकरण है



$y - 0 = \frac{0+4}{3-1}(x-3)$

$y = 2x - 6$ .....(1)

बिन्दु 'C' , (1) तथा $x + y = 5$ का प्रतिच्छेद बिन्दु है

$\therefore$ $C\left(\frac{11}{3}, \frac{4}{3}\right)$

अब B(3, 0) का रेखा $x + y = 5$ के सापेक्ष प्रतिबिम्ब

$$\frac{x-3}{1} = \frac{y-0}{1} = -2\left(\frac{3+0-5}{1+1}\right)$$

$x = 5, y = 2$

$\therefore$ B' (5, 2)

परिवर्तित रेखा CD का समीकरण

$y - 2 = \frac{\frac{4}{3}-2}{\frac{11}{3}-5}(x-5) \Rightarrow 2(y-2) = (x-5) \Rightarrow x - 2y = 1$

$\therefore$ $D\left(0, \frac{-1}{2}\right)$.

8. AB is a variable line sliding between the co-ordinate axes in such a way that A lies on X-axis and B lies on Y-axis. If P is a variable point on AB such that PA = b, PB = a and AB = a + b, then equation of locus of P is

(A*) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) $x^2 - y^2 = a^2 + b^2$

AB एक चर रेखा है जो निर्देशी अक्षों के बीच इस प्रकार फिसल रही है कि A, X-अक्ष पर स्थित है और B, Y-अक्ष पर स्थित है। यदि AB पर एक चर बिन्दु P इस प्रकार है कि PA = b, PB = a और AB = a + b, तो P के बिन्दुपथ का समीकरण है–

(A*) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) $x^2 - y^2 = a^2 + b^2$

Sol. By geometry

$\alpha^2 + \beta^2 = (a + b)^2$ ....(i)

By section formula

$h = \frac{\alpha a}{a+b} \Rightarrow \alpha = \frac{h(a+b)}{a}$

$k = \frac{\beta b}{a+b} \Rightarrow \beta = \frac{k(a+b)}{b}$

Put value of α and β in (i)

$\frac{h^2(a+b)^2}{a^2} + \frac{k^2(a+b)^2}{b^2} = (a + b)^2 \Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

Locus of P is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hindi. ज्यामिति से

$\alpha^2 + \beta^2 = (a + b)^2$ ....(i)

विभाजन सूत्र से

$h = \frac{\alpha a}{a+b} \Rightarrow \alpha = \frac{h(a+b)}{a}$

$k = \frac{\beta b}{a+b} \Rightarrow \beta = \frac{k(a+b)}{b}$

समीकरण (i) में α तथा β का मान रखने पर

$\frac{h^2(a+b)^2}{a^2} + \frac{k^2(a+b)^2}{b^2} = (a + b)^2 \Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

P का बिन्दुपथ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ है।

9. If the distance of any point (x, y) from the origin is defined as d(x, y) = max { |x|, |y|}, d(x, y) = a (where 'a' is non-zero constant), then the locus is

(A) A circle (B) Straight line (C*) A square (D) A triangle

यदि किसी बिन्दु (x, y) की मूल बिन्दु से परिभाषित दूरी d(x, y) = max { |x|, |y|}, d(x, y) = a (जहाँ 'a' अशून्य अचर है) तब बिन्दुपथ है

(A) एक वृत्त (B) सरल रेखा (C) एक वर्ग (D) एक त्रिभुज

Sol. d(x, y) = max {|x|, |y|} and d(x, y) = a $\Rightarrow$ a = max. { |x|, |y|}

if यदि |x| > |y| then a = |x| $\Rightarrow$ x = ± a

if यदि |x| < |y| then a = |y| $\Rightarrow$ y = ± a

So, locus of represents a straight line. इसलिए बिन्दुपथ सरल रेखा का व्यक्त करता है।

10. Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed. Then find locus of the foot of the perpendicular drawn from P to AB.

अचर लम्बाई c के एक सरल रेखाखण्ड के दो सिरे A और B स्थिर आयतीय अक्षों क्रमशः OX और OY पर फिसलते है। यदि आयत OAPB पूर्ण किया जाता है, तो P से AB पर डाले गये लम्ब के पाद का बिन्दुपथ है।

(A*) $x^{2/3} + y^{2/3} = c^{2/3}$ (B) $x^{2/3} + y^{2/3} = c^{1/3}$ (C) $x^{1/3} + y^{1/3} = c^{2/3}$ (D) $x^{1/3} + y^{1/3} = c^{1/3}$

Sol. $a^2 + b^2 = c^2$ .... (i)

Let L is (x_1, y_1)
मानाकि L के निर्देशांक (x_1, y_1) है
L is foot of perpendicular from point P(a, b) on line AB
बिन्दु P(a, b) से रेखा AB पर डाले गये लम्बपाद बिन्दु L है।
equation of AB is $bx + ay - ab = 0$ AB का समीकरण $bx + ay - ab = 0$ है।

$\Rightarrow \quad \frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-(ab + ab - ab)}{a^2 + b^2}$ $\Rightarrow \quad \frac{x_1 - a}{b} = \frac{y_1 - b}{a} = \frac{-ab}{c^2}$

$\Rightarrow \quad x_1 = a - \frac{ab^2}{c^2} = \frac{a(c^2 - b^2)}{c^2} = a^3/c^2$

$\Rightarrow \quad a^3 = c^2 x_1$ .... (ii)

similarly इसी प्रकार

$b^3 = c^2 y_1$ ..... (iii)

using these relations (ii) & (iii) in equation (i), we get required locus.
(ii) व (iii) का समीकरण (i) में प्रयोग करने पर अभीष्ठ बिन्दु–पथ ज्ञात होता है।

11. Let the line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the x and y axes at A and B respectively. Now a line parallel to the given line cuts the coordinate axis at P and Q and points P and Q are joined to B and A respectively. The locus of intersection of the joining lines is

माना रेखा $\frac{x}{a} + \frac{y}{b} = 1$, x-अक्ष और y-अक्ष को क्रमश A और B पर प्रतिच्छेद करती है अब दी गई रेखा के समान्तर रेखा, निर्देशांक अक्षों को P और Q पर मिलती है तथा बिन्दुओं P और Q को B और A से क्रमशः जोड़ा जाता है तब दी गई रेखाओं के प्रतिच्छेद बिन्दु का बिन्दुपथ है –

(A*) $\frac{x}{a} - \frac{y}{b} = 0$ (B) $\frac{x}{a} + \frac{y}{b} = 0$ (C) $\frac{x}{b} - \frac{y}{a} = 0$ (D) $\frac{x}{b} + \frac{y}{a} = 0$

Sol. Equation of the line PQ is $\frac{x}{a} + \frac{y}{b} = \lambda$

Now equation of PB is

$y = -\frac{b}{a\lambda}(x) + b$

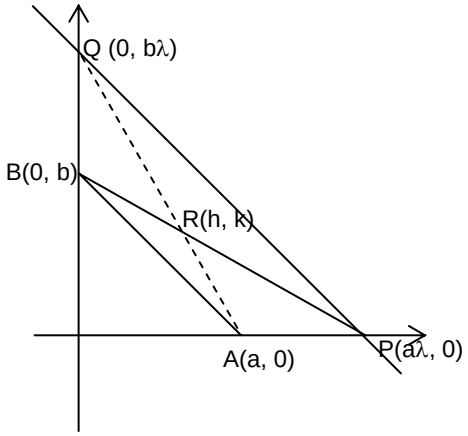
$a\lambda y + bx = \lambda ab$ …(1)

Also equation of the line AQ is

$ay + b\lambda x = ab\lambda$ ….(2)

For point of intersection solving (1) and (2), we get

$h = \frac{a\lambda}{\lambda + 1}$, $k = \frac{b\lambda}{\lambda + 1}$.



Eliminating λ we get, $\frac{h}{a}=\frac{k}{b}$ or $\frac{h}{a}-\frac{k}{b}=0$

Hence locus is $\frac{x}{a}-\frac{y}{b}=0$.

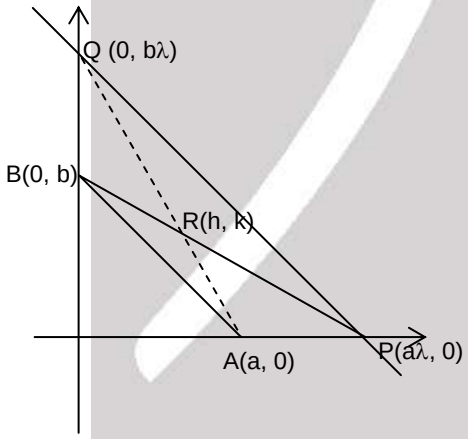
Hindi रेखा PQ का समीकरण $\frac{x}{a}+\frac{y}{b}=\lambda$ है।

अब PB का समीकरण $y = -\frac{b}{a\lambda}(x)+b$ है

$a\lambda y + bx = \lambda ab$ …(1)

तथा रेखा AQ का समीकरण $ay + b\lambda x = ab\lambda$ है ….(2)

(1) और (2) को हल करने पर प्रतिच्छेद बिन्दु $h=\frac{a\lambda}{\lambda+1}$, $k=\frac{b\lambda}{\lambda+1}$.



λ को विलोपित करने पर, $\frac{h}{a}=\frac{k}{b}$ या $\frac{h}{a}-\frac{k}{b}=0$

अतः बिन्दुपथ $\frac{x}{a}-\frac{y}{b}=0$ है।

12. A variable line whose slope is –2 cuts the x and y axes respectively at points A and C. A rhombus ABCD is completed such that vertex B lies on the line y = x. Then the locus of vertex D is

एक चर रेखा जिसकी प्रवणता –2 है क्रमशः x और y अक्ष को बिन्दुओं A और C पर मिलती है। एक समचतुर्भुज ABCD पूर्ण इस प्रकार किया गया है कि शीर्ष B, रेखा y = x. पर स्थित है तब शीर्ष D का बिन्दु पथ है –

(A) $2x + y = 1$ (B) $x - y = 0$ (C*) $x + y = 0$ (D) $x + 2y = 0$

Sol. Let vertex D be (h, k).

Equation of line AC is $y = -2x + \lambda$ … (1)

therefore point A and C are $\left(\frac{\lambda}{2}, 0\right)$ and $(0, \lambda)$

since ABCD is a rhombus therefore AC and BD are mutually perpendicular.

$\Rightarrow$ equation of BD is $x - 2y + \frac{3\lambda}{4} = 0$ … (2)

since B lies on $y = x \Rightarrow B = (x_1, x_1)$ … (3)

from (2) and (3)

$\Rightarrow B = \left(\frac{3\lambda}{4}, \frac{3\lambda}{4}\right)$

since E is the mid point of both AC and BD

hence $\frac{h + \frac{3\lambda}{4}}{2} = \frac{\lambda}{4} \Rightarrow h = -\frac{\lambda}{4}$ … (4)

and $\frac{k + \frac{3\lambda}{4}}{2} = \frac{\lambda}{2} \Rightarrow k = \frac{\lambda}{4}$ … (5)

from (4) + (5),

$\Rightarrow h + k = 0 \Rightarrow x + y = 0$

Hence locus of D is $x + y = 0$.

Hindi माना शीर्ष D (h, k) है।

रेखा AC का समीकरण $y = -2x + \lambda$ है। … (1)

चूंकि A और C के निर्देशांक $\left(\frac{\lambda}{2}, 0\right)$ और $(0, \lambda)$ है।

चूंकि ABCD एक समचतुर्भुज है इसलिए AC और BD परस्पर लम्बवत् है –

$\Rightarrow$ BD का समीकरण $x - 2y + \frac{3\lambda}{4} = 0$ है। … (2)

चूंकि B रेखा $y = x \Rightarrow B = (x_1, x_1)$ पर स्थित है। … (3)

(2) और (3) से

$\Rightarrow B = \left(\frac{3\lambda}{4}, \frac{3\lambda}{4}\right)$

चूंकि E, AC और BD का मध्य बिन्दु है।

अतः $\frac{h + \frac{3\lambda}{4}}{2} = \frac{\lambda}{4} \Rightarrow h = -\frac{\lambda}{4}$ … (4)

और $\frac{k + \frac{3\lambda}{4}}{2} = \frac{\lambda}{2} \Rightarrow k = \frac{\lambda}{4}$ … (5)

4) + (5) से

$\Rightarrow h + k = 0 \Rightarrow x + y = 0$

अतः बिन्दु D का बिन्दुपथ $x + y = 0$ है।

13. ABCD is a square away from origin of side length 'a'. Its side AB slides between x and y–axes in first quadrant with A on x-axis and B on y-axis. The locus of the foot of perpendicular dropped from the point E on the diagonal AC (where E is the midpoint of the side AD), is
ABCD एक 'a' लम्बाई की भुजा का वर्ग है। इसकी भुजा AB, x और y अक्ष के मध्य प्रथम चतुर्थांश में फिसलता है, बिन्दु E से विकर्ण है AC पर डाले गए लम्ब के लम्बपाद का बिन्दुपथ है। (जहा E भुजा AD का मध्य बिन्दु है)

(A) $(y-x)^2+(x-3y)^2=a^2$ (B) $(y-x)^2+(x-3y)^2=\frac{a^2}{2}$

(C*) $(y-x)^2+(x-3y)^2=\frac{a^2}{4}$ (D) None of these इनमें से कोई नहीं

Sol. Let vertex A slides on y–axis and vertex B slides on x–axis coordinates of the point A are (0, a sin θ) and that of C are (a cos θ + a sin θ, a cos θ)

In ΔAEF, $AF=\frac{a}{2}\cos 45^0=\frac{a}{2\sqrt{2}}$

And $FC=AC-AF=\sqrt{2}a-\frac{a}{2\sqrt{2}}=\frac{3a}{2\sqrt{2}}$

$\Rightarrow AF:FC=\frac{a}{2\sqrt{2}}:\frac{3a}{2\sqrt{2}}=1:3$ **[2]**

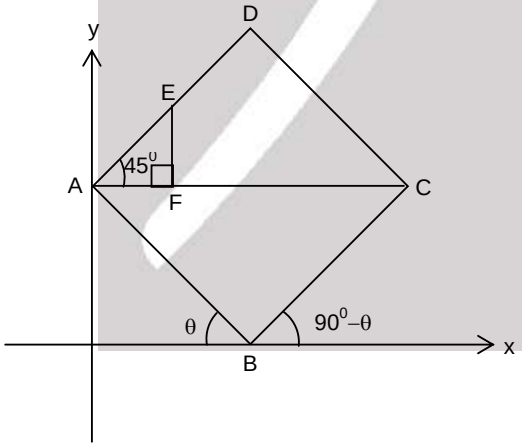
$\Rightarrow$ Let the coordinates of the point F are (x, y)

$\Rightarrow x=\frac{3\times 0+1(a\cos\theta+a\sin\theta)}{4}=\frac{a(\sin\theta+\cos\theta)}{4}$

$\Rightarrow \frac{4x}{a}=\sin\theta+\cos\theta$ … (1)

and $y=\frac{3a\sin\theta+a\cos\theta}{4}$

$\Rightarrow \frac{4y}{a}=3\sin\theta+\cos\theta$ … (2)



From (1) and (2),

$\sin\theta=\frac{2(y-x)}{a}$ and $\cos\theta=\frac{6x-2y}{a}$

$\Rightarrow (y-x)^2+(3x-y)^2=\frac{a^2}{4}$ is the locus of the point F. **[1]**

Hindi माना शीर्ष A, y अक्ष पर फिसलता है तथा शीर्ष B, x–अक्ष पर फिसलता है बिन्दु A के निर्देशांक

(0, a sin θ) तथा C के (a cos θ + a sin θ, a cos θ)

ΔAEF में AF = $\frac{a}{2}\cos 45^0 = \frac{a}{2\sqrt{2}}$

और FC = AC – AF = $\sqrt{2}a - \frac{a}{2\sqrt{2}} = \frac{3a}{2\sqrt{2}}$

$\Rightarrow$ AF : FC = $\frac{a}{2\sqrt{2}} : \frac{3a}{2\sqrt{2}}$ = 1 : 3 **[2]**

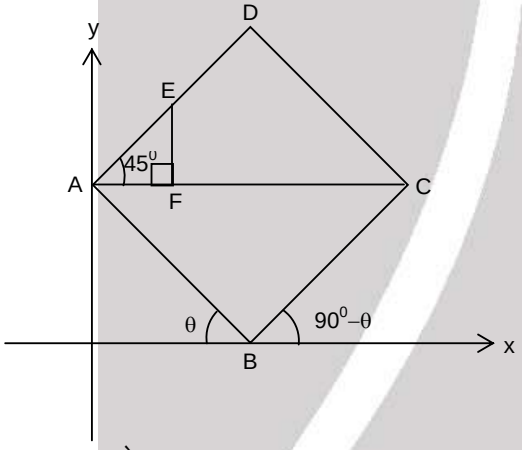
$\Rightarrow$ माना बिन्दु F के निर्देशांक (x, y) है।

$\Rightarrow x = \frac{3\times 0 + 1(a\cos\theta + a\sin\theta)}{4} = \frac{a(\sin\theta + \cos\theta)}{4}$

$\Rightarrow \frac{4x}{a} = \sin\theta + \cos\theta$ … (1) **[1]**

और $y = \frac{3a\sin\theta + a\cos\theta}{4}$

$\Rightarrow \frac{4y}{a} = 3\sin\theta + \cos\theta$ … (2) **[1]**



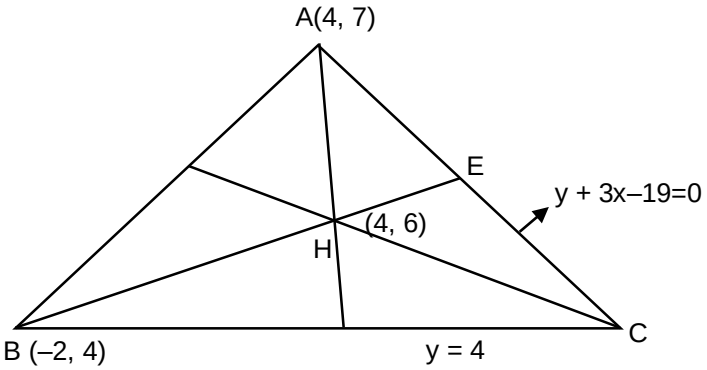
(1) व (2) से

$\sin\theta = \frac{2(y-x)}{a}$ and $\cos\theta = \frac{6x-2y}{a}$

$\Rightarrow (y-x)^2 + (3x-y)^2 = \frac{a^2}{4}$ बिन्दु F का बिन्दुपथ है।

14_. Let orthocentre of ΔABC is (4, 6). If A = (4, 7) and B = (–2, 4), the find coordinates of vertex C
माना ΔABC का लम्बकेन्द्र (4, 6) है यदि A = (4, 7) तथा B = (–2, 4) है तब शीर्ष C के निर्देशांक है।
(A*) (5, 4) (B) (4, 5) (C) (–5, –4) (D) (–4, –5)

Ans.

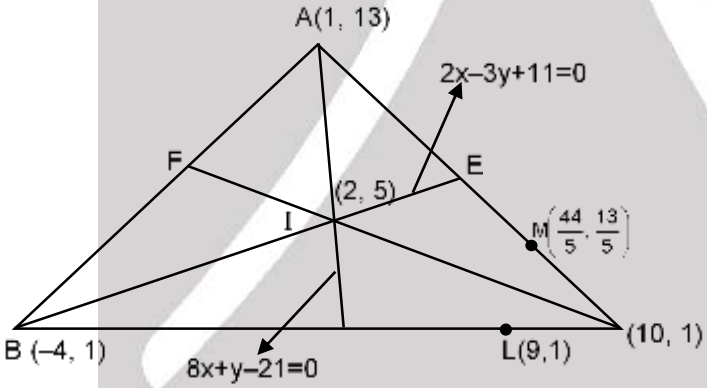


Let orthocentre is H(4, 6)
Equation of AH is x = 4
Equation of BC is y = 4
Slope of BH is 1/3
Slope of AC is –3
equation of AC is y + 3x – 19 = 0
$\Rightarrow$ C is (5, 4)

15_. Let Incentre of ΔABC is (2, 5). If A = (1, 13) and B = (–4, 1), then find coordinates of C

(A) (1, 10) (B*) (10, 1) (C) (8, 2) (D) (9, 3)

Sol.



Let Incentre is I = (2, 5)
equation of AI is 8x + y – 21 = 0
equation of BI is 2x – 3y + 11 = 0
Reflection of B(–4, 1) about AI is $\left(\frac{44}{5}, \frac{13}{5}\right)$ = M
Reflection of A(1, 13) about BI is (9, 1) = L
equation of BL is y = 1
equation of AM is 4x + 3y – 43 = 0 $\Rightarrow$ C is (10, 1)
माना अन्तकेन्द्र I = (2, 5)

AI का समीकरण 8x + y – 21 = 0 है।

BI का समीकरण 2x – 3y + 11 = 0 है।

B(–4, 1) क्ष AI में प्रतिबिम्ब $\left(\frac{44}{5}, \frac{13}{5}\right)$ = M है।

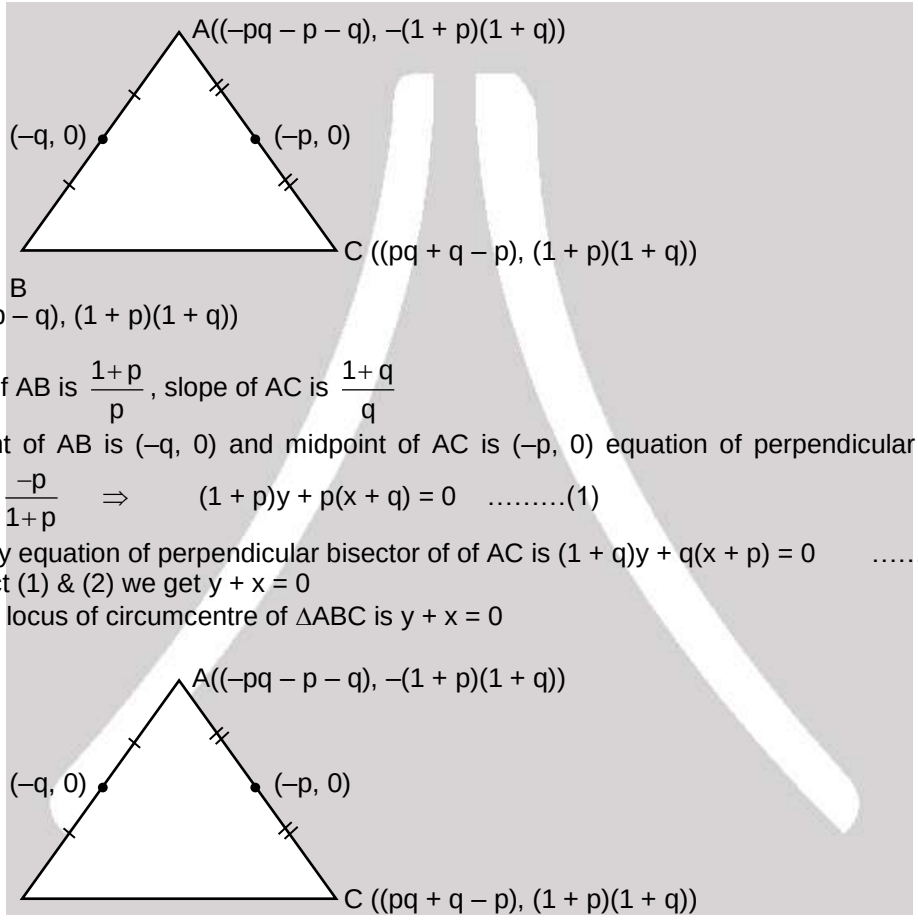
A(1, 13) का BI मे प्रतिबिम्ब (9, 1) = L है।

BL का समीकरण y = 1 है।

AM का समीकरण 4x + 3y – 43 = 0 है। $\Rightarrow$ C is (10, 1)

16. The locus of circumcentre of the triangle formed by vertices A((–pq – p – q), –(1 + p)(1 + q)), B(pq + p – q, (1 + p)(1 + q)), C(pq + q – p, (1 + p)(1 + q)) is
त्रिभुज जिसके शीर्ष A((–pq – p – q), –(1 + p)(1 + q)), B(pq + p – q, (1 + p)(1 + q)), C(pq + q – p, (1 + p)(1 + q)) है, के परिकेन्द्र का बिन्दुपथ है।
(A*) y + x = 0 (B) y – x = 0 (C) $x^2 + y^2 = 1$ (D) xy = 1

Sol.



Slope of AB is $\frac{1+p}{p}$, slope of AC is $\frac{1+q}{q}$

Midpoint of AB is (–q, 0) and midpoint of AC is (–p, 0) equation of perpendicular bisector of AB is

$\frac{y-0}{x+q} = \frac{-p}{1+p}$ $\Rightarrow$ (1 + p)y + p(x + q) = 0 .....(1)

Similarly equation of perpendicular bisector of of AC is (1 + q)y + q(x + p) = 0 .....(2)
Subtract (1) & (2) we get y + x = 0
$\Rightarrow$ locus of circumcentre of ΔABC is y + x = 0

Hindi.

AB की प्रवणता $\frac{1+p}{p}$, AC की प्रवणता $\frac{1+q}{q}$ है।

AB के मध्य बिन्दु (–q, 0) है तथा AC का मध्य बिन्दु बिन्दु (–p, 0) है AB के लम्ब समद्विभाजक का समीकरण है

$\frac{y-0}{x+q} = \frac{-p}{1+p}$ $\Rightarrow$ (1 + p)y + p(x + q) = 0 .....(1)

इसी प्रकार AC के लम्ब समद्विभाजक का समीकरण है (1 + q)y + q(x + p) = 0 .....(2)
(1) मे से (2) घटाने पर y + x = 0
$\Rightarrow$ ΔABC के परिकेन्द्र का बिन्दुपथ y + x = 0 है।

17. Let two sides of rectangle of area 20 units are along lines $x - y = 0$ and $x + y = 2$, then the locus of point of intersection of diagnals is
(A) $(x-1)^2 + (y-1)^2 = 10$ or $(y-1)^2 + (x-1)^2 = 10$
(B*) $(x-1)^2 - (y-1)^2 = 10$ or $(y-1)^2 - (x-1)^2 = 10$
(C) $(x+1)^2 - (y+1)^2 = 10$ or $(y+1)^2 - (x+1)^2 = 10$
(D) $(x-1)^2 + (y-1)^2 = 10$ or $(y+1)^2 - (x+1)^2 = 10$
माना 20 इकाई क्षेत्रफल के आयत की दो भुजाएँ रेखाओं $x - y = 0$ और $x + y = 2$ के अनुदिश है तब विकर्णों के प्रतिच्छेद बिन्दु का बिन्दु का बिन्दुपथ है–
(A) $(x-1)^2 + (y-1)^2 = 10$ या $(y-1)^2 + (x-1)^2 = 10$
(B*) $(x-1)^2 - (y-1)^2 = 10$ या $(y-1)^2 - (x-1)^2 = 10$
(C) $(x+1)^2 - (y+1)^2 = 10$ या $(y+1)^2 - (x+1)^2 = 10$
(D) $(x-1)^2 + (y-1)^2 = 10$ या $(y+1)^2 - (x+1)^2 = 10$

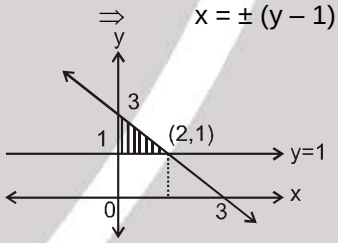
Sol. $\left|\frac{h-k}{\sqrt{2}}\right| \times \left|\frac{h+k-2}{\sqrt{2}}\right| = \frac{1}{4} \times 20$

$\Rightarrow |h - k|\,|h + k - 2| = 10$
$\Rightarrow |(h-1) - (k-1)|\,|(h-1) + (k-1)| = 10$
$\Rightarrow (h-1)^2 - (k-1)^2 = 10$ or $(k-1)^2 - (h-1)^2 = 10$
$\Rightarrow (x-1)^2 - (y-1)^2 = 10$ or $(y-1)^2 - (x-1)^2 = 10$

18. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is
(A*) 2 sq units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units
रेखा युग्म $x^2 - y^2 + 2y = 1$ के कोण समद्विभाजकों एवं रेखा $x + y = 3$ से निर्मित त्रिभुज का क्षेत्रफल है –

(A) 2 वर्ग इकाई (B) 4 वर्ग इकाई (C) 6 वर्ग इकाई (D) 8 वर्ग इकाई

Sol. $x^2 - y^2 + 2y = 1 \quad \Rightarrow \quad x = \pm (y - 1)$



Bisector of above lines are $x = 0$, $y = 1$ उपरोक्त रेखाओं के समद्विभाजक है $x = 0$ व $y = 1$
so Area between $x = 0$, $y = 1$ and $x + y = 3$
अतः $x = 0$, $y = 1$ तथा $x + y = 3$ के मध्य क्षेत्रफल $= \frac{1}{2} \times 2 \times 2 = 2$ squ. units वर्ग इकाई

19. Equation of the line pair through the origin and perpendicular to the line pair $xy - 3y^2 + y - 2x + 10 = 0$ is :
रेखा युग्म $xy - 3y^2 + y - 2x + 10 = 0$ केलम्बवत् एवं मूल बिन्दु से गुजरने वाले रेखा युग्म का समीकरण है :

(A) $xy - 3y^2 = 0$ (B*) $xy + 3x^2 = 0$ (C) $xy + 3y^2 = 0$ (D) $x^2 - y^2 = 0$

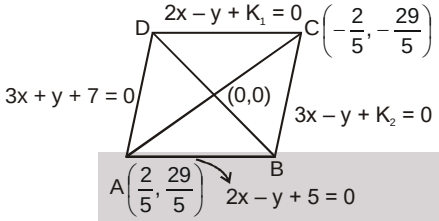
Sol. Let equations of lines represented by the line pair $xy - 3y^2 + y - 2x + 10 = 0$ are $y + c_1 = 0$, $x - 3y + c_2 = 0$
माना रेखा युग्म $xy - 3y^2 + y - 2x + 10 = 0$ से निरूपित रेखाएँ $y + c_1 = 0$, $x - 3y + c_2 = 0$ हैं।
lines $\perp$ to these lines and passing through origin are
उपरोक्त रेखाओं के लम्बवत् रेखाऐं जोकि मूलबिन्दु से गुजरती है
$x = 0$, $y = -3x$
Joint equation संयुक्त समीकरण
$x(3x + y) = 0 \quad \Rightarrow \quad xy + 3x^2 = 0$

20. Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.
दो सरल रेखाओं के समीकरण ज्ञात कीजिए जो दी गई रेखाओं $6x^2 - xy - y^2 + x + 12y - 35 = 0$ से समान्तर चतुर्भुज बनाती है जिसके विकर्ण मूल बिन्दु पर प्रतिच्छेद करते है

(A*) $6x^2 - xy - y^2 - x - 12y - 35 = 0$ (B) $6x^2 - xy - y^2 - x - 12y + 35 = 0$
(C) $6x^2 - xy - y^2 - x + 12y - 35 = 0$ (D) $6x^2 - xy - y^2 + x - 12y - 35 = 0$

Sol. $6x^2 - xy - y^2 + x + 12y - 35 = 0 \quad \Rightarrow \quad (2x - y + 5)(3x + y - 7) = 0$
Now point A is intersection point of $2x - y + 5 = 0$ and $3x + y - 7$

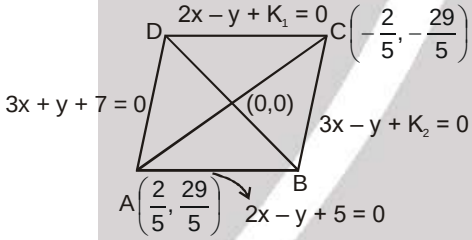
$\therefore \quad A\left(\frac{2}{5}, \frac{29}{5}\right)$



Now $C\left(-\frac{2}{5}, -\frac{29}{5}\right)$ as midpoint of diagonals is (0, 0)
$2x - y + K_1 = 0$ and $3x + y + K_2 = 0$ are passing through C
$\therefore \quad K_1 = -5$, and $K_2 = 7$
Now combined equation $(2x - y - 5)(3x + y + 7) = 0$
$6x^2 - xy - y^2 - x - 12y - 35 = 0$

Sol. $6x^2 - xy - y^2 + x + 12y - 35 = 0 \quad \Rightarrow \quad (2x - y + 5)(3x + y - 7) = 0$
अब बिन्दु A रेखाओं $2x - y + 5 = 0$ और $3x + y - 7$ का प्रतिच्छेद बिन्दु है

$\therefore \quad A\left(\frac{2}{5}, \frac{29}{5}\right)$



अब $C\left(-\frac{2}{5}, -\frac{29}{5}\right)$ विकर्ण का मध्य बिन्दु (0, 0) है
$2x - y + K_1 = 0$ तथा $3x + y + K_2 = 0$ बिन्दु C से गुजरती है
$\therefore K_1 = -5$ तथा $K_2 = 7$
संयुक्त समीकरण $(2x - y - 5)(3x + y + 7) = 0$
$6x^2 - xy - y^2 - x - 12y - 35 = 0$

21. The curve passing through the points of intersection of $S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ and $S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines which are

(A*) equally inclined to the x - axis (B) perpendicular to each other
(C) parallel to each other (D) Not equally inclined to y-axis

$S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$ एवं $S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ के प्रतिच्छेद बिन्दुओं से गुजरनें वाला वक्र एक सरल रेखा युग्म को प्रदर्शित करता है जो—

(A*) x-अक्ष से समान कोंण पर झुकी है। (B) परस्पर लम्बवत है।
(C) एक दुसरे के समान्तर (D) y-अक्ष से समान नहीं झुकी हुई है।

Sol. Curve passing through points of intersection of $S_1 = 0$ & $S_2 = 0$ is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right) + \lambda(x^2 + y^2 + 2gx + 2fy + c) = 0$$

above equation represents a pair of straight lines. They are parallel to the lines $\frac{x^2}{a^2}+\frac{y^2}{b^2} + \lambda(x^2 + y^2) = 0$ which represents a pair of lines equally inclined to axis as the term containing xy is absent

Hindi. $S_1 = 0$ एवं $S_2 = 0$ के प्रतिच्छेद बिन्दुओं से गुजरने वाले वक्र का समीकरण

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right) + \lambda(x^2 + y^2 + 2gx + 2fy + c) = 0 \text{ है।}$$

उपरोक्त समीकरण सरल रेखा युग्म को दर्शाता है। ये सरल रेखाओं $\frac{x^2}{a^2}+\frac{y^2}{b^2} + \lambda(x^2 + y^2) = 0$ के समान्तर है जो x-अक्ष से समान कोंण पर झुकी हुई रेखाओं को प्रदर्शित करती है क्योंकि इसमें xy वाला पद विद्यमान नहीं है।

PART-II: NUMERICAL VALUE QUESTIONS

भाग-II : संख्यात्मक प्रश्न (NUMERICAL VALUE QUESTIONS)

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

निर्देश :

- इस खण्ड में प्रत्येक प्रश्न का उत्तर **संख्यात्मक मान** के रूप में है जिसमें दो पूर्णांक अंक तथा दो अंक दशमलव के बाद में है।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के **दो** स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।

1. If the points $\left(\frac{a^3}{a-1},\frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1},\frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1},\frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a, b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc– (ab + bc + ac) + 3 (a + b + c).

यदि बिन्दु $\left(\frac{a^3}{a-1},\frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1},\frac{b^2-3}{b-1}\right)$ तथा $\left(\frac{c^3}{c-1},\frac{c^2-3}{c-1}\right)$ तीन भिन्न–भिन्न मानों a, b व c के लिये संरेखीय हो तथा $a \neq 1$, $b \neq 1$ व $c \neq 1$, तो abc– (ab + bc + ac) + 3 (a + b + c) का मान है

Ans. **00.00**

Sol. Let equation of line is $\ell x + my + n = 0$ ...(i)

given $\left(\frac{a^3}{a-1},\frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1},\frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1},\frac{c^2-3}{c-1}\right)$ are collinear

$\left(\frac{t^3}{t-1},\frac{t^2-3}{t-1}\right)$ is general point which satisfies line (i)

$$\ell\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0 \quad \Rightarrow \quad \ell t^3 + m t^2 + nt - (3m + n) = 0$$

$a + b + c = -\frac{m}{\ell} \quad \Rightarrow \quad ab + bc + ac = \frac{n}{\ell} \quad \Rightarrow \quad abc = \frac{3m+n}{\ell}$

Now LHS = abc – (ab + bc + ac) + 3 (a + b + c) = $\frac{(3m+n)}{\ell} - \frac{n}{\ell} + 3\left(\frac{-m}{\ell}\right) = 0$

Hindi माना रेखा का समीकरण $\ell x + my + n = 0$ ...(i)

दिया गया ह $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ संरेखीय है।

$\left(\frac{t^3}{t-1}, \frac{t^2-3}{t-1}\right)$ व्यापक बिन्दु है जो रेखा (i) को संतुष्ट करता है।

$\ell\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0 \quad \Rightarrow \quad \ell t^3 + m t^2 + nt - (3m + n) = 0$

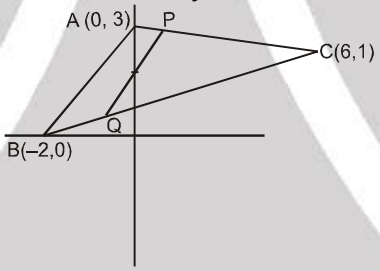
$a + b + c = -\frac{m}{\ell} \quad \Rightarrow \quad ab + bc + ac = \frac{n}{\ell} \quad \Rightarrow \quad abc = \frac{3m+n}{\ell}$

अब LHS = abc – (ab + bc + ac) + 3 (a + b + c) = $\frac{(3m+n)}{\ell} - \frac{n}{\ell} + 3\left(\frac{-m}{\ell}\right) = 0$

2. The complete set of values of λ for which point $(\lambda, \lambda + 1)$ is an interior points of ΔABC, where A ≡ (0, 3), B ≡ (–2, 0) and C ≡ (6, 1) is (p,q) then value of $p^2 + q^2$ is
यदि λ के सभी मानों का समुच्चय जिसके लिए बिन्दु $(\lambda, \lambda + 1)$ त्रिभुज ABC के अंदर स्थित हो जहाँ A ≡ (0, 3), B ≡ (–2, 0) व C ≡ (6, 1) है, (p,q) हो तो $p^2 + q^2$ का मान होगा –

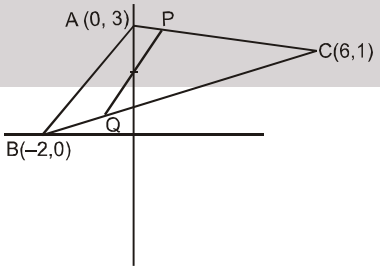
Ans. **02.98**

Sol. Since $(\lambda, \lambda + 1)$ lies on y = x + 1
equation of A B : 3x – 2y + 6 = 0 ; BC : x – 8y + 2 = 0 ; AC : x + 3y – 9 = 0



Line y = x + 1 cuts AC at $P\left(\frac{3}{2}, \frac{5}{2}\right)$ cut BC at $Q\left(\frac{-6}{7}, \frac{1}{7}\right)$. Hence $\lambda \in \left(\frac{-6}{7}, \frac{3}{2}\right)$

Hindi. चूँकिं $(\lambda, \lambda + 1)$, y = x + 1 पर है।
AB की समीकरण : 3x – 2y + 6 = 0 ; BC : x – 8y + 2 = 0 ; AC : x + 3y – 9 = 0



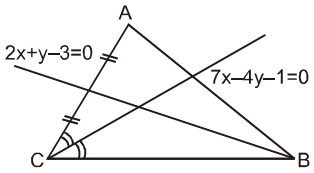
रेखा y = x + 1, AC को बिन्दु $P\left(\frac{3}{2}, \frac{5}{2}\right)$ पर तथा BC को $Q\left(\frac{-6}{7}, \frac{1}{7}\right)$ पर प्रतिच्छेद करती है। अतः $\lambda \in \left(\frac{-6}{7}, \frac{3}{2}\right)$.

3. Let ABC be a triangle such that the coordinates of the vertex A are (– 3, 1). Equation of the median through B is 2x + y – 3 = 0 and equation of the angular bisector of C is 7x – 4y – 1 = 0. Find the slope of line BC.

माना ABC एक त्रिभुज है जिसके शीर्ष A के निर्देशांक (– 3, 1) है। B से जाने वाली माध्यिका का समीकरण 2x + y – 3 = 0 है और C के कोण अर्द्धक का समीकरण 7x – 4y – 1 = 0 है तब रेखा BC की प्रवणता ज्ञात कीजिए।

Ans. 18.00

Sol. Since C lies on 7x – 4y – 1 = 0, therefore let us choose its coordinates as $\left(h, \frac{7h-1}{4}\right)$.



The mid point of AC, i.e. $\left(\frac{h-3}{2}, \frac{7h+3}{8}\right)$ lies on 2x + y – 3 = 0,

therefore we have $\left(\frac{h-3}{2}\right)+\left(\frac{7h+3}{8}\right)-3=0$ gives h = 3

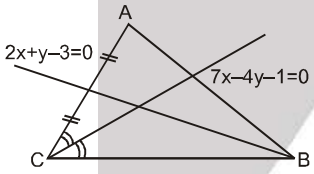
Hence, coordinates of C are (3, 5) and equation of AC is

$$y-5=\frac{5-1}{3+3}(x-3) \quad \text{i.e.,} \quad 2x-3y+9=0 \qquad .....(1)$$

Let slope of BC = m. Since lines BC and AC $\left(\text{slope}=\frac{2}{3}\right)$ are equally inclined to the line 7x – 4y – 1 = 0

$\left(\text{slope}=\frac{7}{4}\right)$, therefore we have i.e., $\frac{m-\frac{7}{4}}{1+\frac{7m}{4}}=\frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{6}}$ (see figure) i.e., $\frac{4m-7}{7m+4}=\frac{1}{2}$ gives m = 18.

Hindi. चूंकि C रेखा 7x – 4y – 1 = 0 पर स्थित है इसलिए इसके निर्देशांक $\left(h, \frac{7h-1}{4}\right)$.



AC का मध्य बिन्दु $\left(\frac{h-3}{2}, \frac{7h+3}{8}\right)$ रेखा 2x + y – 3 = 0 पर स्थित है

इसलिए $\left(\frac{h-3}{2}\right)+\left(\frac{7h+3}{8}\right)-3=0$ जो h = 3 देता है

अतः C के निर्देशांक (3, 5) और AC का समीकरण है।

$$y-5=\frac{5-1}{3+3}(x-3) \quad \text{अथवा} \quad 2x-3y+9=0 \qquad .....(1)$$

माना BC की प्रवणता = m है चूंकि रेखा BC और AC $\left(\text{प्रवणता}=\frac{2}{3}\right)$ रेखा 7x – 4y – 1 = 0 से बराबर झुकी हुई है।

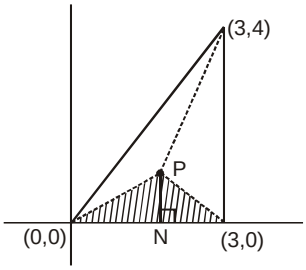
$\left(\text{प्रवणता}=\frac{7}{4}\right)$ इसलिए $\frac{m-\frac{7}{4}}{1+\frac{7m}{4}}=\frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{6}}$ (चित्र से), $\frac{4m-7}{7m+4}=\frac{1}{2}\Rightarrow$ m = 18 देता है।

4. A(3, 4), B(0, 0) and C(3, 0) are vertices of ΔABC. If 'P' is the point inside the ΔABC, such that d(P, BC) ≤ min. {d(P, AB), d (P, AC)}. Then the maximum of d (P, BC) is.
(where d(P, BC) represent distance between P and BC).
ΔABC के शीर्ष A(3, 4), B(0, 0) और C(3, 0) है। यदि ΔABC का आन्तरिक बिन्दु 'P' इस प्रकार है कि d(P, BC) ≤ min. {d(P, AB), d (P, AC)} तब d (P, BC) का अधिकतम मान है

(जहाँ d(P, BC), P और BC के मध्य दूरी को व्यक्त करता है।).

Ans. **01.00**

Sol. Here BP and CP are angular bisectors. Maximum of d(P, BC) occurs, when P is incentre of ΔABC.
यहाँ BP तथा CP कोण अर्द्धक है। d(P, BC) की अधिकतम दूरी जब P, ΔABC का अन्तःकेन्द्र है



$\therefore$ Maximum of d(P, BC) = PN = ordinate of incentre = 1.
$\therefore$ d(P, BC) की अधिकतम दूरी = PN = अन्तःकेन्द्र की कोटी = 1

5. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line $2x + y = 13$. Then find the area of the triangle.

मूल बिन्दु से दो परस्पर लम्बवत् सरल रेखाएँ, सरल रेखा $2x + y = 13$ के साथ समद्विबाहु त्रिभुज बनाते हुए खींची जाती है, तो त्रिभुज का क्षेत्रफल है –

Ans. **33.80**

Sol. Perpendicular distance from origin to line $2x + y - 13 = 0$ will be

$$p = \left|\frac{0+0-13}{\sqrt{5}}\right| = \frac{13}{\sqrt{5}}$$

$$\tan 45^\circ = \frac{p}{x} \Rightarrow p = x$$

$$\text{Hence area} = \frac{1}{2}(2x)(p) = px = p^2 = \frac{169}{5}$$

Hindi. मूल बिन्दु की सरल रेखा $2x + y - 13 = 0$ से लम्बवत् दूरी होगी

$$p = \left|\frac{0+0-13}{\sqrt{5}}\right| = \frac{13}{\sqrt{5}}$$

$$\tan 45^\circ = \frac{p}{x} \Rightarrow p = x$$

$$\text{अतः क्षेत्रफल} = \frac{1}{2}(2x)(p) = px = p^2 = \frac{169}{5}$$

6. On the straight line $y = x + 2$, a point (a, b) is such that the sum of the square of distances from the straight lines $3x - 4y + 8 = 0$ and $3x - y - 1 = 0$ is least, then find value of (a + b).
सरल रेखा $y = x + 2$ पर एक बिन्दु (a, b) इस प्रकार है कि बिन्दु की रेखाओं $3x - 4y + 8 = 0$ और $3x - y - 1 = 0$ से दुरियों के वर्गो का योगफल न्यूनतम है तब (a + b) का मान ज्ञात कीजिए.

Ans. **04.72 or 04.73**

Sol. Point be (x, y) but it lies on $y = x + 2$ So, (x, x + 2)

$$F(x) = \left[\frac{3x-4(x+2)+8}{\sqrt{3^2+4^2}}\right]^2 + \left[\frac{3x-(x+2)-1}{\sqrt{3^2+1^2}}\right]^2 = \frac{2x^2+5[4x^2-12x+9]}{50} = \frac{22\left[\left(x-\frac{30}{22}\right)^2-\frac{900}{484}\right]+45}{50}$$

F(x) is minimum at $x = \frac{15}{11}$. So point is $\left(\frac{15}{11}, \frac{37}{11}\right) = (a, b)$

$11 (a + b) = 52.$

Hindi. मानाकि बिन्दु (x, y) परन्तु यह $y = x + 2$ पर स्थित है इसलिए (x, x + 2)

$$F(x)=\left[\frac{3x-4(x+2)+8}{\sqrt{3^2+4^2}}\right]^2+\left[\frac{3x-(x+2)-1}{\sqrt{3^2+1^2}}\right]^2=\frac{2x^2+5[4x^2-12x+9]}{50}=\frac{22\left[\left(x-\frac{30}{22}\right)^2-\frac{900}{484}\right]+45}{50}$$

F(x) x = $\frac{15}{11}$ पर न्यूनतम है इसलिए बिन्दु $\left(\frac{15}{11},\frac{37}{11}\right)$ = (a, b) है।

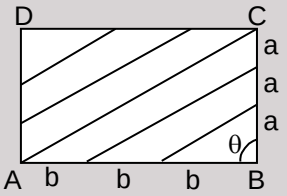
11 (a + b) = 52.

7. Parallelogram ABCD is cut by (2n –1) number of parallel lines in which one is diagnal AC. Distance between any two nearest lines is same which is also equal to distance of B, D from respective nearest line anong these. Ratio of area of smallest triangle so formed to area of parallelogram is $\frac{1}{32}$. Find n.

Ans. **04.00**

समान्तर चतुर्भुज ABCD को (2n –1) समान्तर रेखाओं से काटा जाता है जिसमें से एक रेखा विकर्ण AC है। दो नजदीक रेखाओं के मध्य दुरी बराबर है तथा B, D इस रेखाओं मे से नजदीक रेखा की दुरी के बराबर है इस प्रकार बने सबसे छोटे त्रिभुज का समान्तर चतुर्भुज के क्षेत्रफल का अनुपात $\frac{1}{32}$ है तब n ज्ञात कीजिए।

Sol. $\frac{\frac{1}{2}ab\times\sin\theta}{na\times nb\times\sin\theta}=\frac{1}{32}$ $\Rightarrow$ $n^2=16\Rightarrow$ $n=4$



8. A is a variable point on x-axis and B(0,b) is a fixied point. A equilateral triangle ABC is completed with vertex C away from origin. If the locus of the point C is $\alpha x + \beta y = b$, then $\alpha^3 + \beta^3$ is

एक चर बिन्दु A, x अंक्ष B(0,b) नियत बिन्दु है एक समबाहु त्रिभुज ABC , मूल बिन्दु से दूर शीर्ष C के द्वारा पूर्ण किया जाता है। यदि बिन्दु C का बिन्दुपथ $\alpha x + \beta y = b$ हो तो

$\alpha^3 + \beta^3$ का मान है –

Ans. **04.19 or 04.20**

Sol. Let माना $\angle OAB = \theta$

and और AB = AC = b cosec θ.

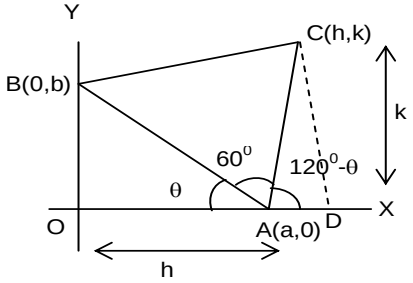
Since चूंकि $h = OA + AD = OA + AC\cos(120^0 - \theta)$

$\Rightarrow h = a + b\,\text{cosec}\theta\cos(120^0 - \theta)$

$\Rightarrow h = b\cot\theta + b\,\text{cosec}\theta\cos(120^0 - \theta)$.

(since चूंकि in Δ OAB, $a = b\cot\theta$)

$$\Rightarrow h=\frac{b}{2}\left(\cot\theta+\sqrt{3}\right). \qquad \ldots (1)$$



Also $k = AD\sin(120^0 - \theta)$

$\Rightarrow k = b\cos ec\theta \cdot \sin\left(120^0 - \theta\right) = \frac{b}{2}\left(\sqrt{3}\cot\theta + 1\right)$ ... (2)

from (1) and और (2) से

$\frac{2h}{b} - \sqrt{3} = \left(\frac{2k}{b} - 1\right)\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h - k = b$

Hence locus of अतः C का बिन्दु पथ is $\sqrt{3}x - y = b$ है।

9. The portion of the line $ax + by - 1 = 0$, intercepted between the lines $ax + y + 1 = 0$ and $x + by = 0$ subtends a right angle at the origin and the condition in a and b is $\lambda a + b + b^2 = 0$, then find value of λ.
रेखाओं $ax + y + 1 = 0$ और $x + by = 0$ के मध्य अन्तखण्डित रेखा $ax + by - 1 = 0$ का भाग मूल बिन्दु पर समकोण बनाती है तथा a और b में प्रतिबन्ध $\lambda a + b + b^2 = 0$ है तब λ का मान ज्ञात कीजिए।

Ans. **02.00**

Sol. Given lines are $ax + y + 1 = 0$ .....(i)
$x + by = 0$ .....(ii)
$ax + by = 1$ .....(iii)
Joint equation of (i) and (ii) is $(ax + y + 1)(x + by) = 0$
$\Rightarrow$ $ax^2 + by^2 + (ab + 1)xy + x + by = 0$
Making (iv) homogeneous with the help of equation (i) we have
$ax^2 + by^2 + (ab + 1)xy + x(ax + by) + by(ax + by) = 0$
since angle between these two lines is 90^0
$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$
$2a + b + b^2 = 0$ is the required condition.

Hindi. दी गई रेखाएं $ax + y + 1 = 0$ .....(i)
$x + by = 0$ .....(ii)
$ax + by = 1$ .....(iii)
(i) और (ii) की सहायता से $(ax + y + 1)(x + by) = 0$
$\Rightarrow$ $ax^2 + by^2 + (ab + 1)xy + x + by = 0$
(iv) को (i) की सहायता से समघात बनाने पर
$ax^2 + by^2 + (ab + 1)xy + x(ax + by) + by(ax + by) = 0$
चूंकि दोनों रेखाओं के मध्य कोण 90^0 है।
$\therefore$ x^2 का गुणांक + y^2 का गुणांक = 0
$2a + b + b^2 = 0$ अभीष्ट प्रतिबन्ध है।

10. If the straight lines joining the origin and the points of intersection of the curve $5x^2 - 24xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis, then find the value of $|k|$.
वक्र $5x^2 - 24xy - 6y^2 + 4x - 2y + 3 = 0$ एवं रेखा $x + ky - 1 = 0$ के प्रतिच्छेद बिन्दुओं को मूल बिन्दु से मिलाने वाली सरल रेखाएँ x-अक्ष से समान कोण पर झुकी हुई हो, तो $|k|$ का मान है–

Ans. 02.60

Sol. Homogenize $5x^2 - 24xy - 6y^2 + 4x - 2y + 3 = 0$ by $x + ky = 1$
$5x^2 - 24xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$
it is equally indined with x-axes hence coeff. $xy = 0 \Rightarrow -24 + 4k - 2 + 6k = 0$
$\Rightarrow k = \frac{26}{10} = 02.60$

Hindi. $5x^2 - 24xy - 6y^2 + 4x - 2y + 3 = 0$ को $x + ky = 1$ की सहायता से समघात बनाने पर
$5x^2 - 24xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$
ये रेखाऐं x-अक्ष से समान कोण पर झुकी है अतः xy का गुणांक $= 0 \Rightarrow -24 + 4k - 2 + 6k = 0$
$\Rightarrow k = \frac{26}{10} = 02.60$

11. If the points of intersection of curves $C_1 = 4y^2 - 2\lambda x^2 - 2xy - 9x + 3$ and

$C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ subtends a right angle at origin, then find the value of λ.

यदि वक्रों $C_1 = 4y^2 - 2\lambda x^2 - 2xy - 9x + 3$ एवं $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ के प्रतिच्छेद बिन्दु मूल बिन्दु पर समकोण आन्तरित करते हैं, तो λ का मान है :

Ans. **09.50**

Sol. Equation of family of curves passing through intersection of C_1 & C_2 is

$-2\lambda x^2 + 4y^2 - 2xy - 9x + 3 + \mu(2x^2 + 3y^2 - 4xy + 3x - 1) = 0$ .....(i)

It will give the joint equation of pair of lines passing through origin,

if coefficient of x = 0 & Constant = 0 $\Rightarrow$ $\mu = 3$

put $\mu = 3$ in equation (i), we get

$-2\lambda x^2 + 4y^2 - 2xy + 6x^2 + 9y^2 - 12xy = 0$

It will subtend 90º at origin if coeff. of x^2 + coeff. of $y^2 = 0 \Rightarrow \lambda = \frac{19}{2}$

Hindi. C_1 और C_2 के प्रतिच्छेद बिन्दुओं से गुजरने वाले वक्र निकाय का समीकरण

$2\lambda x^2 + 4y^2 - 2xy - 9x + 3 + \mu(2x^2 + 3y^2 - 4xy + 3x - 1) = 0$ .....(i)

यह मूल बिन्दु से गुजरने वाले रेखा युग्म को निरूपित करेगा यदि x का गुणांक = 0 तथा अचर पद = 0

$\Rightarrow \mu = 3$

$\mu = 3$ समीकरण (i) में रखने पर

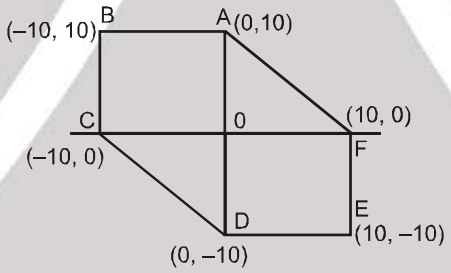
$-2\lambda x^2 + 4y^2 - 2xy + 6x^2 + 9y^2 - 12xy = 0$

यह मूल बिन्दु पर समकोण अन्तरित करेगा यदि x^2 का गुणांक + y^2 का गुणांक = 0

$\lambda + 6 + 13 = 0 \Rightarrow \lambda = \frac{19}{2}$

12. Let the integral points inside or on the boundary of region bounded by straight lines as shown in figure is equal to k, then $\sqrt{k}$ is equal to

चित्र में दर्शायेनुसार सरल रेखाओं से परिबद्ध क्षेत्र में अथवा रेखाओं पर स्थित पूर्णांक बिन्दुओं की संख्या k के बराबर है तब $\sqrt{k}$ बराबर है–



Ans. 18.19

(0, 0) (0, 1) ..... (0, 10) $\rightarrow$ 11

(–1, 0) (–1, 1) ..... (–1, 10) $\rightarrow$ 11 11^2

(–10, 0) (–10, 1) (–10, 2), (–10, 10) $\rightarrow$ 11

Similarly ODEF = 11^2

origin is common to both $\Rightarrow$ integral point in

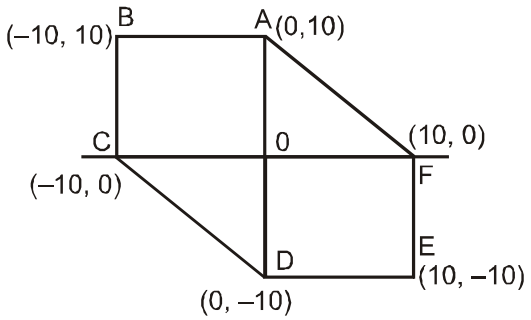
region O ABC and ODEF = $11^2 + 11^2 - 1 = 241$ ..... (1)

consider region OAF excluding OA & OF

(1, 9), (1, 8) ....(1, 1) $\rightarrow$ 9

(2, 8), (2, 7) ....(2, 1) $\rightarrow$ 8

(9, 1) $\rightarrow$ 1



= total points 1 + 2 + ... + 8 + 9 = $\frac{9\times10}{2}$ = 45 points ..... (2)

Similarly region OCD = 45 points

Total integral points = 241 + 45 + 45 = 331 ..... (3)

Hindi. क्षेत्र OABC के लिये x-निर्देशांक नीचे की ओर 0 से –10 तक परिवर्तित होता है।

(0, 0) (0, 1) ..... (0, 10) → 11

(–1, 0) (–1, 1) ..... (–1, 10) → 11 11^2

(–10, 0) (–10, 1) (–10, 2), (–10, 10) → 11

इसी प्रकार ODEF = 11^2

दोनो मे मूल बिन्दु उभयनिष्ट है

⇒ क्षेत्र OABC एवं ODEF में पूर्णांक बिन्दुओं की संख्या

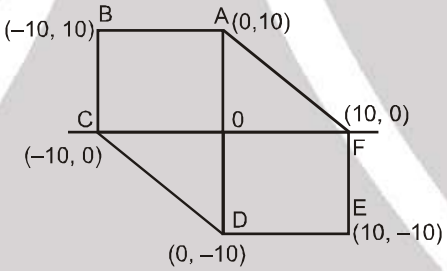
= $11^2 + 11^2 - 1 = 241$ ..... (1)

क्षेत्र OAF में OA एवं OF को छोड़ते हुए

(1, 9), (1, 8) ....(1, 1) → 9

(2, 8), (2, 7) ....(2, 1) → 8

(9, 1) → 1



= कुल बिन्दु 1 + 2 + ... + 8 + 9 = $\frac{9\times10}{2}$ = 45 बिन्दु ..... (2)

इसी प्रकार क्षेत्र OCD में = 45 बिन्दु होंगे ..... (3)

कुल पूर्णांक बिन्दु = 241 + 45 + 45 = 331

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

भाग - III : एक या एक से अधिक सही विकल्प प्रकार

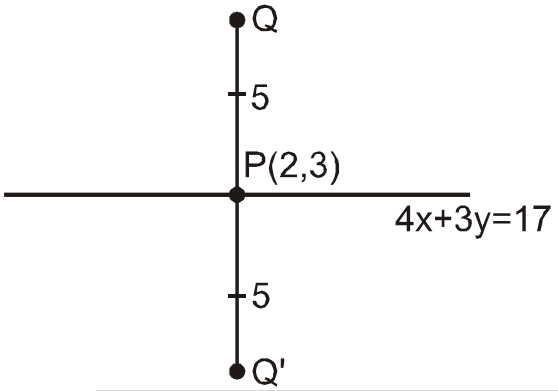
1. Point P(2, 3) lies on the line 4x + 3y = 17. Then find the co-ordinates of points farthest from the line which are at 5 units distance from the P.

बिन्दु P(2, 3) रेखा 4x + 3y = 17 पर स्थित है तब बिन्दुओं के निर्देशांक ज्ञात कीजिए जो P से 5 इकाई दूरी पर स्थित है तथा इस रेखा से अधिकतम दुरी पर है।

(A*) (6, 6) (B) (6, –6) (C) (2, 0) (D*) (–2, 0)

Sol. For Q and Q' Q और Q' के लिए

$$\frac{x-2}{\frac{4}{5}} = \frac{y-3}{\frac{3}{5}} = \pm 5$$

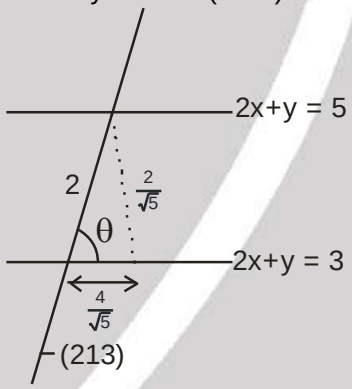


Q(6, 6) and तथा Q'(– 2, 0).

2. Find the equation of the line passing through the point (2, 3) & making intercept of length 2 units between the lines y + 2x = 3 & y + 2x = 5.
बिन्दु (2, 3) से गुजरने वाली तथा रेखाओं y + 2x = 3 तथा y + 2x = 5 के मध्य 2 ईकाई का अंतःखण्ड बनाने वाली रेखा की समीकरण है–

(A) 3x – 4y = 18 (B*) x = 2 (C*) 3x + 4y = 18 (D) x + 2 = 0

Sol. Let slope of line passing through (2, 3) is m. Hence y – 3 = m (x – 2)



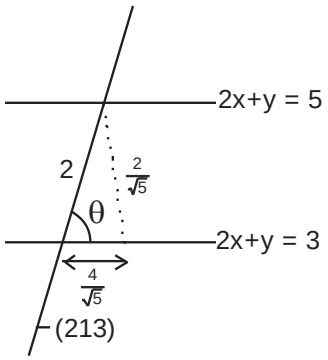
tan θ = ½

Now, $\tan\theta = \left|\frac{m-(-2)}{1+m(-2)}\right| = \frac{1}{2} \Rightarrow \frac{m+2}{1-2m} = \pm\frac{1}{2}$

$\Rightarrow 2m + 4 = \pm(1-2m) \Rightarrow m = -\frac{3}{4}$, not defined

Hence equation of line 3x + 4y = 18, x = 2

Hindi. माना (2, 3) से गुजरने वाली रेखा की ढ़ाल m है अतः
y – 3 = m (x – 2)



$\tan\theta = ½$

अब, $\tan\theta = \left|\frac{m-(-2)}{1+m(-2)}\right| = \frac{1}{2} \Rightarrow \frac{m+2}{1-2m} = \pm\frac{1}{2} \Rightarrow 2m + 4 = \pm(1-2m) \Rightarrow m = -\frac{3}{4}$, परिभाषित नहीं

अतः रेखा की समीकरण $3x + 4y = 18$, $x = 2$

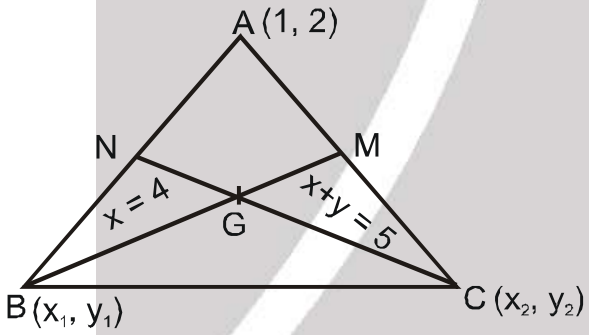
3. In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then the co-ordinates of B and C will be
एक त्रिभुज ABC में शीर्ष A के निर्देशांक (1, 2) तथा B एवं C से जाने वाली माध्यिकाँऐ क्रमशः $x + y = 5$ तथा $x = 4$ है, तो B व C के निर्देशांक होंगे –

(A) $(-2, 7), (4, 3)$ (B*) $(7, -2), (4, 3)$

(C) $(2, 7), (-4, 3)$ (D) $(2, -7), (3, -4)$

Sol. $x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)



co - ordinates of G are ≡ (4, 1)

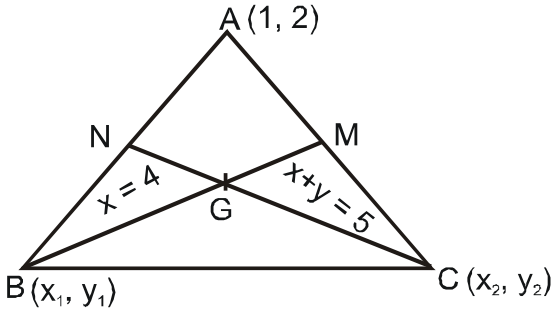
$\Rightarrow \quad \frac{1+x_1+x_2}{3} = 4$ ....(iii)

and $\frac{y_1+y_2+2}{3} = 1$ ... (iv)

solving above equations, we get B & C.

Hindi. $x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)



G के निर्देशांक ≡ (4, 1)

$\Rightarrow \quad \frac{1+x_1+x_2}{3} = 4$ ....(iii)

तथा $\frac{y_1+y_2+2}{3} = 1$ ... (iv)

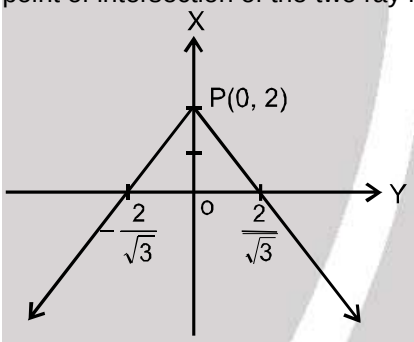
उपरोक्त समीकरणों को हल करके B व C के निर्देशांक प्राप्त होंगे।

4. A is a point on either of two rays $y + \sqrt{3}\,|x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co–ordinates of the foot of perpendicular from A on the bisector of the angle between them is/are

$y + \sqrt{3}\,|x| = 2$ द्वारा प्रदर्शित दो किरणों में से किसी किरण पर उनके प्रतिच्छेद बिन्दु से $\frac{4}{\sqrt{3}}$ दूरी पर बिन्दु A स्थित है। इन किरणों के मध्य के कोण अर्द्धक पर A से डाले गए लम्ब के पाद के निर्देशांक है –

(A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (B*) (0, 0) (C) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (D) (0, 4)

Sol. ∵ point of intersection of the two ray is P(0, 2)

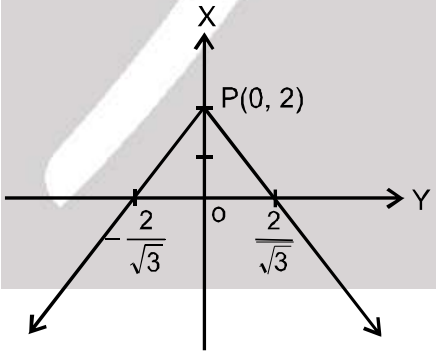


∴ Point A is $\left(\frac{2}{\sqrt{3}}, 0\right)$ or $\left(-\frac{2}{\sqrt{3}}, 0\right)$

and PO is bisector of the angle between two rays

∴ required point is (0, 0)

Hindi ∵ दी गई किरणों का प्रतिच्छेद बिन्दु P(0, 2) है



∴ बिन्दु A ≡ $\left(\frac{2}{\sqrt{3}}, 0\right)$ या $\left(-\frac{2}{\sqrt{3}}, 0\right)$

चूँकि PO दी गई किरणों के मध्य के कोण का अर्घक है

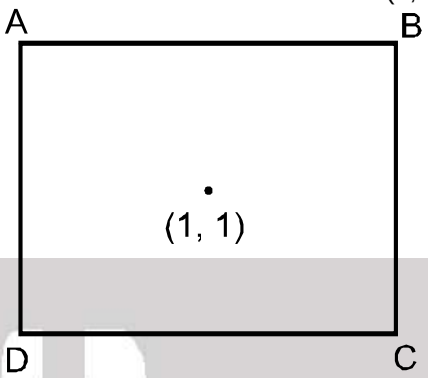
∴ इसलिए अभिष्ट बिन्दु (0, 0)

5. If one side of a square is parallel to $3x - 4y = 0$ & its area being 16 while centre being (1, 1), then find equation of sides of square.
यदि वर्ग की एक भुजा, रेखा $3x - 4y = 0$ के समान्तर तथा इसका क्षेत्रफल 16 है। जबकि केन्द्र (1, 1) है। तब वर्ग की भुजाओं के समीकरण है–

(A*) $3x - 4y + 11 = 0$ (B*) $3x - 4y - 9 = 0$ (C*) $4x + 3y + 3 = 0$ (D*)$4x + 3y - 17 = 0$

Sol. To find equations of AB and CD

$\because$ AB and CD are parallel to $3x - 4y = 0$ and at a distance of 2 units from (1, 1)



$\therefore$ $3x - 4y + k = 0$ and $\left|\frac{3-4+k}{5}\right| = 2$

$\Rightarrow$ $k - 1 = \pm 10$ $\Rightarrow$ $k = 11, -9$

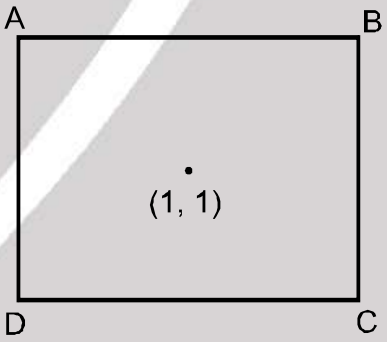
$\therefore$ equations of two sides of the square which are parallel to $3x - 4y = 0$ are
$3x - 4y + 11 = 0$ and $3x - 4y - 9 = 0$
Now the remaining two sides will be perpendicular to
$3x - 4y = 0$ and at a distance of 2 unit from (1, 1)

$\therefore$ $4x + 3y + k = 0$ and $\left|\frac{4+3+k}{5}\right| = 2 \Rightarrow k + 7 = \pm 10 \Rightarrow k = 3, -17$

$\therefore$ remaining two sides are $4x + 3y + 3 = 0$ and $4x + 3y - 17 = 0$

Hindi. AB तथा CD की समीकरण,

$\because$ रेखा $3x - 4y = 0$ के समान्तर तथा बिन्दु (1, 1) से 2 इकाई की दूरी पर स्थित रेखाओं से ज्ञात कर सकतें हैं।



$\therefore$ $3x - 4y + k = 0$ और $\left|\frac{3-4+k}{5}\right| = 2$

$\Rightarrow$ $k - 1 = \pm 10$ $\Rightarrow$ $k = 11, -9$

$\therefore$ अतः वर्ग की दो भुजाओं की समीकरण जोकि $3x - 4y = 0$ के समानान्तर है, है।
$3x - 4y + 11 = 0$ तथा $3x - 4y - 9 = 0$
अब बची हुई दो भुजाएँ $3x - 4y = 0$ के लम्बवत् तथा बिन्दु (1, 1) से 2 इकाई दूरी पर है।

$\therefore$ $4x + 3y + k = 0$ और $\left|\frac{4+3+k}{5}\right| = 2 \Rightarrow k + 7 = \pm 10$ $\Rightarrow$ $k = 3, -17$

$\therefore$ अतः बची हुई दो भुजाएँ हैं– $4x + 3y + 3 = 0$ और $4x + 3y - 17 = 0$

6. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

उस त्रिभुज की भुजाओं की समीकरण ज्ञात कीजिऐ जिसका एक शीर्ष (4, –1) है तथा जिसके दो आंतरिक कोण समद्विभाजकों की समीकरणें $x - 1 = 0$ तथा $x - y - 1 = 0$ है।

(A*) $2x - y + 3 = 0$ (B) $x + 2y - 6 = 0$ (C*) $2x + y - 7 = 0$ (D*) $x - 2y - 6 = 0$

Sol.

Reflection of (4, – 1) about $x = 1$ is B and about $x - y = 1$ is C

बिन्दु (4, – 1) का, रेखा $x = 1$ के सापेक्ष प्रतिबिम्ब B है तथा रेखा $x - y = 1$ के सापेक्ष C है।

$B \equiv (x_1, y_1)$

$C \equiv (x_2, y_2)$

$\left(\frac{x_1+4}{2}, \frac{y_1-1}{2}\right)$ will lie on $x = 1$ बिन्दु $\left(\frac{x_1+4}{2}, \frac{y_1-1}{2}\right)$, रेखा $x = 1$ पर स्थित है

$\Rightarrow \quad \frac{x_1+4}{2} = 1 \quad \Rightarrow \quad x_1 = -2$ and तथा $\frac{y_1+1}{x_1-4} = 0 \quad \Rightarrow \quad y_1 = -1$

$\therefore \quad B \equiv (-2, -1) \quad \Rightarrow \quad \left(\frac{x_2+4}{2}, \frac{y_2-1}{2}\right)$ will lie on $x - y - 1 = 0$

बिन्दु $\left(\frac{x_2+4}{2}, \frac{y_2-1}{2}\right)$, रेखा $x - y - 1 = 0$ पर स्थित है

$x_2 + 4 - y_2 + 1 - 2 = 0 \quad \Rightarrow \quad x_2 - y_2 + 3 = 0$ ..... (i)

तथा $\left(\frac{y_2+1}{x_2-4}\right) . 1 = -1 \quad \Rightarrow \quad x_2 + y_2 - 3 = 0$ .... (ii)

from (i) & व (ii) से $x_2 = 0, y_2 = 3$

$\therefore C(0, 3)$ $\therefore$ Equation of BC $\therefore$ BC का समीकरण

$y + 1 = \frac{4}{2}(x + 2) \quad \Rightarrow \quad 2x - y + 3 = 0$

For finding PQ & PR we have to find Q and R ;

PQ व PR ज्ञात करने के लिए हमें Q व R ज्ञात करने होंगे

solving QI & BC, we get Q (1, 5) QI व BC को हल करने पर Q (1, 5)

equation of PQ is $2x + y - 7 = 0$ PQ का समीकरण $2x + y - 7 = 0$ है।

For R, we solve RI & BC R के लिए RI व BC को हल करने पर

$\therefore$ R (–4, – 5) $\therefore$ R (–4, – 5)

equation of PR is $x - 2y - 6 = 0$ PR का समीकरण $x - 2y - 6 = 0$

7. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then the correct statement(s) among the following is/are (O is origin)

(A) The absolute minimum value of OP + OQ, where O is origin is $18\sqrt{2}$

(B*) Minimum area of ΔOPQ is 32

(C*) The absolute minimum value of OP + OQ, where O is origin is 18

(D*) Area of ΔOPQ is minimum for slope $\left(\frac{-1}{4}\right)$.

ऋणात्मक प्रवणता की एक सरल रेखा L बिन्दु (8, 2) से गुजरती है तथा धनात्मक निर्देशांक अक्षों को P तथा Q पर काटती है। तब निम्न में से कौनसे सही कथन है (जहाँ O मूल बिन्दु है।)

(A) OP + OQ का न्यूनतम निरपेक्ष मान $18\sqrt{2}$ है जहाँ O मूलबिन्दु है।

(B*) ΔOPQ का न्यूनतम क्षेत्रफल 32 है।

(C*) OP + OQ का निरपेक्ष मान 18 है जहाँ O मूलबिन्दु है।

(D*) ΔOPQ के क्षेत्रफल न्यूनतम के लिए प्रवणता $\left(\frac{-1}{4}\right)$ है।

Sol. Equation of line

(A) $y - 2 = m(x - 8)$ and $m < 0$ $\Rightarrow$ $P\left(8 - \frac{2}{m}, 0\right)$ and $Q(0, 2 - 8m)$

$$OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m| = 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18.$$

Area of ΔOPQ is minimum when (8,2) is midpoint of line. So, P(16, 0), Q(0,4) $\Delta OPQ = \frac{1}{2}(16)(4) = 32$.

Sol. रेखा का समीकरण

(A) $y - 2 = m(x - 8)$ और $m < 0$ $\Rightarrow$ $P\left(8 - \frac{2}{m}, 0\right)$ और $Q(0, 2 - 8m)$

$$OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m| = 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18.$$

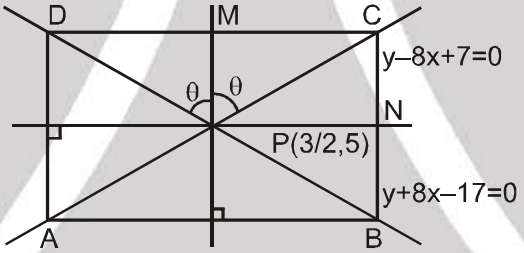
ΔOPQ का क्षेत्रफल न्यूनतम होगा जब रेखा का मध्य बिन्दु (8,2) है।

इसलिए P(16, 0), Q(0,4) $\Delta OPQ = \frac{1}{2}(16)(4) = 32$ है।

8. The equation of the diagonals of a rectangle are $y + 8x - 17 = 0$ and $y - 8x + 7 = 0$. If the area of the rectangle is 8 sq. units, find the equation of the sides of the rectangle.
आयत के विकर्णो की समीकरण $y + 8x - 17 = 0$ और $y - 8x + 7 = 0$ है। यदि आयत का क्षेत्रफल 8 वर्ग इकाई है तो आयत की भुजाओं के समीकरण ज्ञात कीजिए।

(A*) $y = 1$ (B*) $y = 9$ (C*) $x = 1$ (D*) $x = 2$.

Sol. The intersection point of the given diagonals $y + 8x - 17 = 0$ and $y - 8x + 7 = 0$ is $P \equiv \left(\frac{3}{2}, 5\right)$.



Now, we have $\Delta CPD = \frac{1}{4}$ (ABCD) = 2sq. units.

i.e., $\ell^2 \sin\theta \cos\theta = 2$ [putting CP = ℓ] i.e., $\ell^2 \sin 2\theta = 4$ .....(i)

Where $\tan 2\theta = \left|\frac{8 - (-8)}{1 + (8)(-8)}\right| = \frac{16}{63}$ therefore $\sin 2\theta = \frac{16}{65}$ and $\cos 2\theta = \frac{63}{65}$

Putting in equation (1), we have $\ell^2 = \frac{65 \times 4}{16} = \frac{65}{4}$ i.e., $\ell = \frac{\sqrt{65}}{2}$.

Therefore, we have $PM = \ell \cos\theta = \ell\sqrt{\frac{1 + \cos 2\theta}{2}} = \frac{\sqrt{65}}{2} \times \sqrt{\frac{65 + 63}{2 \times 65}} = 4$

and $PN = \ell \sin\theta = \ell\sqrt{\frac{1 - \cos 2\theta}{2}} = \frac{\sqrt{65}}{2} \times \sqrt{\frac{65 - 63}{2 \times 65}} = \frac{1}{2}$

Equation of the angular bisectors of the diagonals are

$$\frac{y + 8x - 17}{\sqrt{65}} = \pm \frac{y + 8x - 17}{\sqrt{65}} \quad \text{i.e.,} \quad x = \frac{3}{2} \text{ and } y = 5.$$

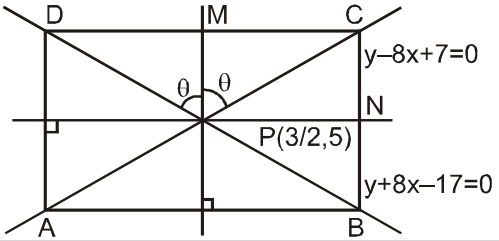
From the figure we can see the sides AB, CD are lines parallel to the angular bisector PN = y – 5 = 0 at a distance of PN = 4 units. Hence, then equations are

$y = 5 \pm 4$ $y = 1, y = 9$.

and the sides AD, BC are lines parallel to the angular bisector PM = $x - \frac{3}{2} = 0$ at a distance of PN = $\frac{1}{2}$ units. Hence, their equations are

$x = \frac{3}{2} \pm \frac{1}{2}$ i.e., $x = 1, x = 2$.

HINDI. दिए गए विकर्णो $y + 8x - 17 = 0$ और $y - 8x + 7 = 0$ is $P \equiv \left(\frac{3}{2}, 5\right)$ का प्रतिच्छेद बिन्दु है–



अब यहाँ $\Delta CPD = \frac{1}{4}$ (ABCD) = 2 वर्ग इकाई

अर्थात $\ell^2 \sin\theta \cos\theta = 2$ [CP = ℓ रखने पर]अर्थात , $\ell^2 \sin 2\theta = 4$ .....(i)

जहाँ $\tan 2\theta = \left|\frac{8-(-8)}{1+(8)(-8)}\right| = \frac{16}{63}$ इसलिए $\sin 2\theta = \frac{16}{65}$ और $c \cos 2\theta = \frac{63}{65}$

समीकरण (1) में रखने पर $\ell^2 = \frac{65 \times 4}{16} = \frac{65}{4}$ अर्थात $\ell = \frac{\sqrt{65}}{2}$.

इसलिए $PM = \ell \cos\theta = \ell\sqrt{\frac{1+\cos 2\theta}{2}} = \frac{\sqrt{65}}{2} \times \sqrt{\frac{65+63}{2\times 65}} = 4$

and $PN = \ell \sin\theta = \ell\sqrt{\frac{1-\cos 2\theta}{2}} = \frac{\sqrt{65}}{2} \times \sqrt{\frac{65-63}{2\times 65}} = \frac{1}{2}$

विकर्णो के कोण अर्द्वकों के समीकरण है–

$\frac{y+8x-17}{\sqrt{65}} = \pm \frac{y+8x-17}{\sqrt{65}}$ अर्थात $x = \frac{3}{2}$ और $y = 5$.

चित्र से भुजाएं AB, CD रेखाओं के समान्तर कोण अर्द्वक है। PN = y – 5 = 0, PN = 4 इकाई पर . अतः समीकरणें है।

$y = 5 \pm 4$ $y = 1, y = 9$.

भुजाएं AD, BC कोण अर्द्वकों के समान्तर है PM = x = 0, PN की दुरी = $\frac{1}{2}$ इकाई.

अतः उनकी समीकरणें है– $x = \frac{3}{2} \pm \frac{1}{2}$ अर्थात, $x = 1, x = 2$.

9. Two adjacent sides of a rhombus are $2x + 3y = a - 5$ and $3x + 2y = 4 - 2a$ and its diagonals intersect at the point (1, 2), then a can be –
समचतुर्भुज की दो आसन्न भुजाऐं $2x + 3y = a - 5$ और $3x + 2y = 4 - 2a$ है तथा इसके विकर्ण बिन्दु (1, 2), पर प्रतिच्छेद करते है तब a हो सकता है

(A*) – 16 (B) 16 (C) $-\frac{10}{3}$ (D*) $\frac{10}{3}$

Sol. We know diagonals are angle bisectors of a rhombus
समचतुर्भुज के कोण अर्द्धक विकर्ण होते है।

$\Rightarrow \left|\frac{2.1+3.2-a+5}{\sqrt{13}}\right| = \left|\frac{3.1+2.2-4+2a}{\sqrt{13}}\right|$

$\Rightarrow |13 - a| = |3 + 2a| \Rightarrow 13 - a = \pm(3 + 2a)$. $\Rightarrow a = -16, \frac{10}{3}$.

10. A line $L_1 \equiv 3y - 2x - 6 = 0$ is rotated about its point of intersection with y–axis in clockwise direction to make it L_2 such that the area formed by L_1, L_2, x–axis and line $x = 5$ is $\frac{49}{3}$ sq units if its point of intersection with $x = 5$ lies below x–axis then points lying on the equation of L_2 are

एक रेखा $L_1 \equiv 3y - 2x - 6 = 0$ को y–अक्ष को इसके प्रतिच्छेद बिन्दु के सापेक्ष दक्षिणावर्त्त दिशा के सापेक्ष घुमाया जाता है। जो इसे L_2 रेखा की दिशा में है इस प्रकार है कि L_1, L_2, x–अक्ष और रेखा $x = 5$ से परिबद्ध क्षेत्रफल $\frac{49}{3}$ वर्ग ईकाई है यदि इसके $x = 5$ के साथ प्रतिच्छेद बिन्दु, x–अंक्ष के नीचे स्थित है तब L_2 रेखा के समीकरण पर स्थित बिन्दु है।

(A*) (3, – 1) (B) (4,2) (C*) (1,1) (D) (3,3)

Sol. $L_1 \equiv 3y - 2x - 6 = 0$

Point about which line rotated is बिन्दु जो रेखा के सापेक्ष घुमाया जाता $A \equiv (0, 2)$ है।

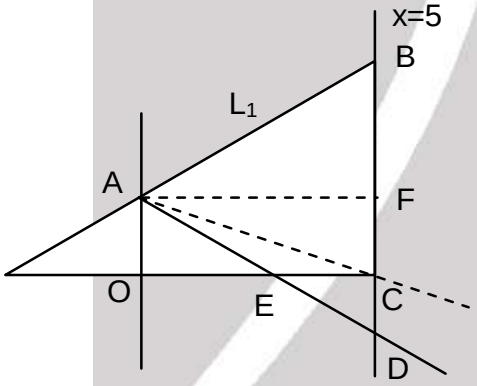
Let equation of L_2 be $y = mx + 2$

माना L_2 रेखा का समीकरण $y = mx + 2$ है।

As lying L_2 will be cutting line $x = 5$ below x–axis.

$A \equiv (0, 2)$, $b \equiv \left(5, \frac{16}{3}\right)$, $C \equiv (5, 0)$, $D \equiv (5, 5m + 2)$,

$E \equiv \left(\frac{-2}{m}, 0\right)$



Area AECB = area ADB – area ECD

AECB का क्षेत्रफल = (ADB का क्षेत्रफल – (ECD का क्षेत्रफल)

$$\frac{49}{3} = \frac{1}{2} \times BD \times AF - \frac{1}{2} \times EC \times CD$$

$$= \frac{1}{2} \times \left[\frac{16}{3} - (5m + 2)\right] \times 5 - \frac{1}{2} \times \left(5 + \frac{2}{m}\right)[-(5m + 2)]$$

$$\Rightarrow \frac{49}{3} = \frac{110m + 12}{6m} \Rightarrow m = -1$$

$\Rightarrow$ Equation of L_2 का समीकरण is $y + x = 2$ है।

11. Let $D(x_4, y_4)$ be a point such that ABCD is a square & M & P are the midpoints of the sides BC & CD respectively, then

(A*) Ratio of the areas of ΔAMP and the square is 3 : 8

(B) Ratio of the areas of ΔMCP & ΔAMD is 1 : 1

(C*) Ratio of the areas of ΔABM & ΔADP is 1 : 1

(D) Ratio of the areas of the quandrilateral AMCP and the square is 1 : 3

माना $D(x_4, y_4)$ कोई बिन्दु इस प्रकार है कि ABCD एक वर्ग है और M तथा P, क्रमशः भुजाओं BC तथा CD के मध्य बिन्दु है, तो

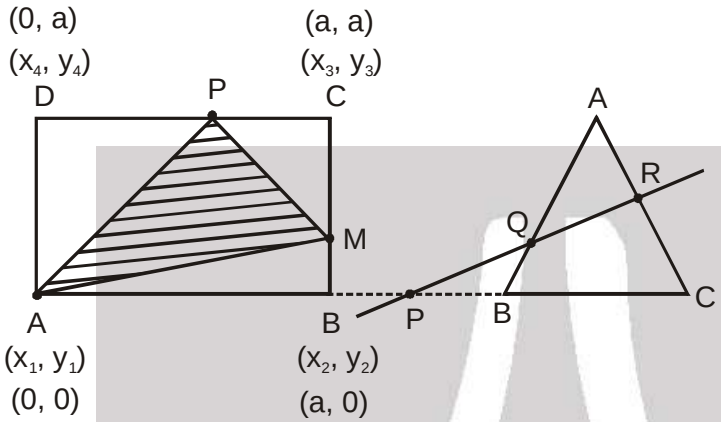
(A*) ΔAMP एवं वर्ग के क्षेत्रफलों का अनुपात 3 : 8 होगा।

(B) ΔMCP तथा ΔAMD के क्षेत्रफलों का अनुपात 1 : 1 होगा।

(C*) ΔABM एवं ΔADP के क्षेत्रफलों का अनुपात 1 : 1 होगा।

(D) चतुर्भुज AMCP एवं वर्ग के क्षेत्रफलों का अनुपात 1 : 3 होगा।

Sol.



Take A(0, 0), B(a, 0), C(a , a) and D(0, a) then M(a, a/2) and P(a/2, a)

$$\Delta AMP = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8} \quad ; \quad \Delta MCP = \frac{a^2}{8} \quad \Rightarrow \quad \Delta ABM = \Delta ADP = \frac{a^2}{4}$$

Area of quad. AMCP = $\frac{3a^2}{8} + \frac{a^2}{8} = \frac{a^2}{2}$

Hindi. माना A(0, 0), B(a, 0), C(a , a) तथा D(0, a) तब M(a, a/2) एवं P(a/2, a)

$$\Delta AMP = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8} \quad ; \quad \Delta MCP = \frac{a^2}{8} \quad \Rightarrow \quad \Delta ABM = \Delta ADP = \frac{a^2}{4}$$

चतुर्भुज AMCP का क्षेत्रफल = $\frac{3a^2}{8} + \frac{a^2}{8} = \frac{a^2}{2}$

12. The equations of perpendicular of the sides AB & AC of Δ ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and circumcenter is $\left(\frac{3}{2}, \frac{5}{2}\right)$, then which of the following is true.

(A) equation of median of side AB is $x - y + 1 = 0$

(B) centroid of triangle ABC is (3, 1)

(C*) vertex C is (4, 0)

(D) Area of triangle ABC is 12.

त्रिभुज ABC की भुजाओं AB व AC के लम्बवत् रेखाओं के समीकरण क्रमशः $x - y - 4 = 0$ तथा $2x - y - 5 = 0$ है। यदि शीर्ष $A(-2, 3)$ हो तथा लम्ब समद्विभाजकों का प्रतिच्छेदन बिन्दु $\left(\frac{3}{2}, \frac{5}{2}\right)$ हो तो निम्न में से कौनसे सही है–

(A) भुजा AB की माध्यिका का समीकरण $x - y + 1 = 0$ है।

(B) त्रिभुज ABC का केन्द्रक (3, 1) है।

(C*) शीर्ष C (4, 0) है।

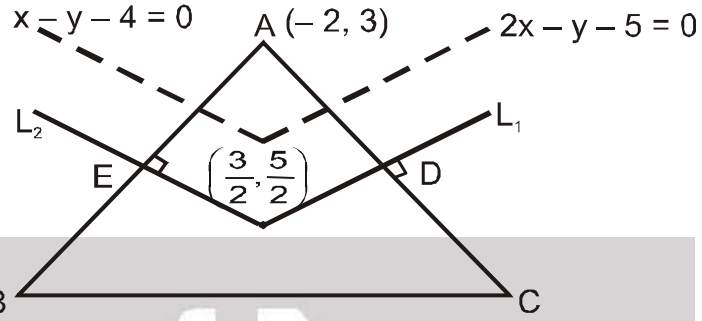
(D) ΔABC का क्षेत्रफल 12 है।.

Sol. Equation of line L_1 is L_1 रेखा का समीकरण $y - \frac{5}{2} = 2 . (x - \frac{3}{2})$

or या $2x - y - \frac{1}{2} = 0$

or या $4x - 2y - 1 = 0$

equation of line L_2 is L_2 रेखा का समीकरण



$y - \frac{5}{2} = 1 (x - \frac{3}{2})$ or या $x - y + 1 = 0$

Point C is mirror image of point A w.r.t line L_1

बिन्दु C रेखा L_1 के सापेक्ष बिन्दु A का दर्पण प्रतिबिम्ब है

$$\frac{x-(-2)}{4} = \frac{y-(3)}{-2} = \frac{-2(-8-6-1)}{20} \quad \therefore \quad C(4, 0)$$

similarly B is mirror image of A in line $L_2 = 0$ इसी प्रकार बिन्दु B, रेखा $L_2 = 0$ के सापेक्ष A का दर्पण प्रतिबिम्ब है

$$\frac{x-(-2)}{1} = \frac{(y-3)}{-1} = \quad \therefore \quad B(2, -1) \quad \Rightarrow \quad D(1, \frac{3}{2}) \ ; E (0, 1)$$

median through B is B से गुजरने वाली माध्यिका $(y + 1) = \frac{5/2}{-1} (x - 2) \Rightarrow 5x + 2y = 8$

median through C is C से गुजरने वाली माध्यिका $(y - 1) = \frac{1}{-4}(x - 0) \Rightarrow x + 4y = 4.$

Area $= \frac{1}{2}\begin{vmatrix} -2 & 3 & 1 \\ 2 & -1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 6$ centroid $\equiv \left(\frac{4}{3}, \frac{2}{3}\right)$.

13. Triangle ABC lies in the cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respecitvely and the coordinates of A are (p, q). The median to the side BC has slope – 5, then which can be corrected.

कार्तीय समतल में त्रिभुज ABC स्थित है जिसका क्षेत्रफल 70 वर्ग इकाई है। B तथा C के निर्देशांक क्रमशः (12, 19) और (23, 20) है और A के निर्देशांक (p, q) है तब भुजा BC की माध्यिका की प्रवणता – 5 है। तब निम्न में से कौनसे सही है–

(A*) $p + q = 47$ (B*) $p + q = 27$ (C) $p - q = 17$ (D*) $p - q = 13$

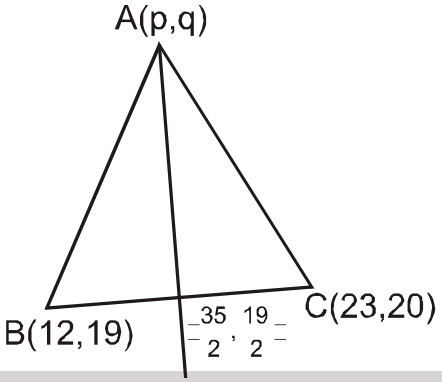
Sol. $\frac{\left(\frac{39}{2}\right) - q}{\left(\frac{35}{2}\right) - p} = -5$ from slope प्रवणता से

$39 - 2q = -5 (35 - 2p)$

$39 - 2q = -175 + 10 p$

i.e., $5p + q = 107$

Also तथा, $\pm \begin{vmatrix} p & q & 1 \\ 12 & 19 & 1 \\ 23 & 20 & 1 \end{vmatrix} = 140 \rightarrow$ From area क्षेत्रफल से



i.e. अर्थात, $11q - p = 337$ $\quad$ $11q - p = 57$

Also तथा, $\underbrace{5p + q = 107}_{\text{solving हल करने पर}}$ $\quad$ $\underbrace{5p + q = 107}_{\text{solving हल करने पर}}$

$\therefore$ $p = 15$ & और $q = 32$ $\quad$ $p = 20$ & और $q = 7$

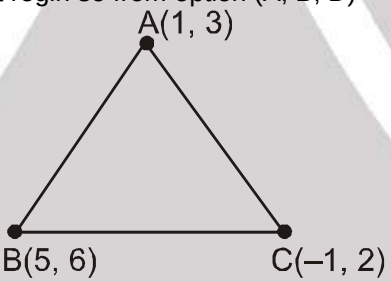
So इसलिए, $p + q = 47$ $\quad$ $p + q = 27$.

14. All the points lying on or inside the triangle formed by the points (1, 3), (5, 6) and (–1, 2) satisfy

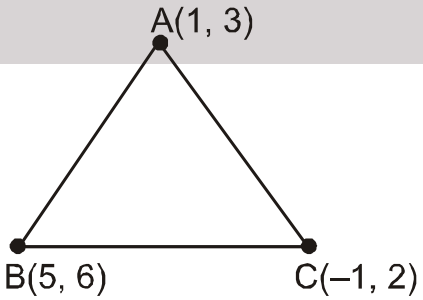
वह सभी बिन्दु (x, y) जो बिन्दुओं (1, 3), (5, 6) तथा (–1, 2) से बनने वाले त्रिभुज पर या त्रिभुज के अन्दर स्थित है, निम्न सम्बन्ध को संतुष्ट करेंगे–

(A*) $3x + 2y \geq 0$ $\quad$ (B*) $2x + y + 1 \geq 0$ $\quad$ (C) $2x + 3y - 12 \geq 0$ $\quad$ (D*) $2x + 11 \geq 0$

Sol. If all three vertex of triangle ABC give same sign for a give line then all the point lying on or inside the ΔABC are on same side for that regin so from option (A, B, D)



Hindi. यदि त्रिभुज ABC के तीन शीर्ष, दी गई रेखा के लिए समान चिन्ह देते है तो सभी बिन्दु ΔABC पर या अन्दर स्थित, क्षेत्र एक ही ओर स्थित होने के लिए विकल्प (A, B, D) से



15. A ≡ (4, 2) and B ≡ (2, 4) are two given points and a point P on the line $3x + 2y + 10 = 0$ is given then which of the following is/are true.

(A*) (PA + PB) is minimum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$

(B) (PA + PB) is maximum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$

(C*) |PA – PB | is maximum when P(– 22, 28)

(D) (PA – PB) is minimum when P(– 22, 28).

A ≡ (4, 2) और B ≡ (2, 4) दो दिए गए बिन्दु है तथा बिन्दु P रेखा 3x + 2y + 10 = 0 पर स्थित है तब निम्न में से कौनसे सत्य है–

(A*) (PA + PB) न्यूनतम है जब $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$ (B) (PA + PB) अधिकतम है जब $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$

(C*) |PA – PB | अधिकतम है जब P(– 22, 28) (D) (PA – PB) न्यूनतम है जब P(– 22, 28).

Ans. (i) $\left(-\frac{14}{5}, -\frac{4}{5}\right)$ (ii) (–22, 28)

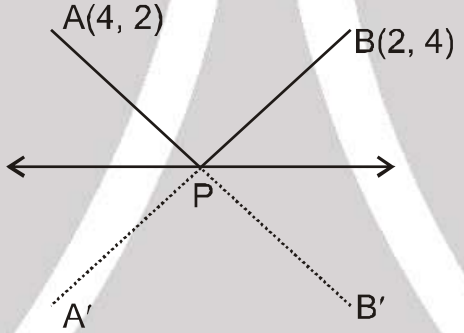
Sol. Since A (4, 2) and B (2,4) both lies same side of 3x + 2y + 10 = 0

(i) PA + PB ≥ AB ; PA + PB′ ≥ AB ⇒ PA + PB = PA + PB′ (min.) = AB

Hence A, P, B′ are collinear.

Image of B(2,4) in 3x + 2y + 10 = 0....is ...(i)

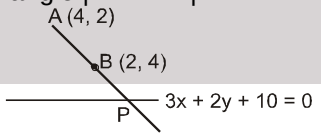
$\frac{x-2}{3} = = -2\left(\frac{6+8+10}{3^2+2^2}\right)$ $\Rightarrow B'(x, y) \equiv \left(-\frac{118}{13}, \frac{-44}{13}\right)$



now equation of AB1 is $y - 2 = \frac{2+\frac{44}{13}}{4+\frac{118}{13}}(x-4) \Rightarrow 7x - 17y + 6 = 0$ .....(ii)

solving (i) and (ii) we get $\left(-\frac{14}{5}, -\frac{4}{5}\right)$

(ii) In any triangle |PA – PB| ≤ AB



Hence |PA – PB| = AB when P, A, B are collinear

Hence equation of AB is y – 2 = – 1 (x – 4)

x + y – 6 = 0 .....(i)

solving (i) with 3x + 2y + 10 = 0 we get (–22, 28)

Hindi. चूँकि A (4, 2) तथा B (2,4) दोनों 3x + 2y + 10 = 0 के एक तरफ है।

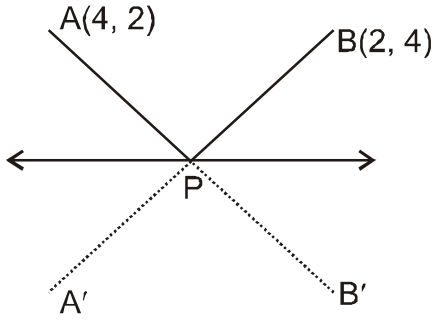
(i) PA + PB ≥ AB ; PA + PB′ ≥ AB

⇒ PA + PB = PA + PB′ (min.) = AB

अतः A, P, B′ संरेखीय है

B(2,4) का रेखा 3x + 2y + 10 = 0...(i) में प्रतिबिम्ब

$$\frac{x-2}{3}=\frac{y-4}{2}=-2\left(\frac{6+8+10}{3^2+2^2}\right) \Rightarrow B'(x,y)\equiv\left(-\frac{118}{13},\frac{-44}{13}\right)$$

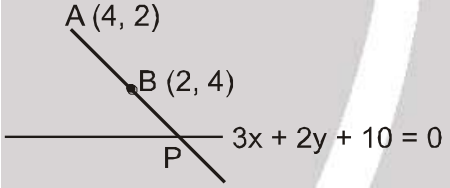


अतः AB1 की समीकरण है $y-2=\frac{2+\frac{44}{13}}{4+\frac{118}{13}}(x-4) \Rightarrow 7x-17y+6=0$ .....(ii)

(i) व (ii) को हल करने पर $\left(-\frac{14}{5},-\frac{4}{5}\right)$

(ii) किसी भी त्रिभुज में $|PA-PB|\le AB$

अतः $|PA-PB|=AB$ जब P, A, B



संरेखीय है।

अतः AB का समीकरण $y-2=-1(x-4)$ है।

$x+y-6=0$ .....(i)

(i) को $3x+2y+10=0$ के साथ हल करने पर हमें प्राप्त होता है (–22, 28)

16. A line passing through $P=(\sqrt{3},0)$ and making an angle of 60° with positive direction of x-axis cuts the parabola $y^2=x+2$ at A and B, then :

एक सरल रेखा बिन्दु $P=(\sqrt{3},0)$ से गुजरती है और धनात्मक x-अक्ष से 60° का कोण बनाती है जो परवलय $y^2=x+2$ को बिन्दु A व B पर काटती है, तो :

(A) $PA+PB=\frac{2}{3}$ (B*) $|PA-PB|=\frac{2}{3}$

(C*) $(PA)(PB)=\frac{4(2+\sqrt{3})}{3}$ (D) $\frac{1}{PA}+\frac{1}{PB}=\frac{2-\sqrt{3}}{2}$

Sol. $\frac{x-\sqrt{3}}{1/2}=\frac{(y-0)}{\sqrt{3}/2}=\pm R \quad \Rightarrow \quad x=\sqrt{3}\pm\frac{R}{2} \quad \& \quad y=\frac{\sqrt{3}}{2}R$

$\Rightarrow \quad \left(\frac{\sqrt{3}}{2}R\right)^2=\sqrt{3}+\frac{R}{2}+2 \Rightarrow \quad \frac{3}{4}R^2=\frac{R}{2}+(2+\sqrt{3}) \quad \Rightarrow \quad 3R^2-2R-4(2+\sqrt{3})=0$

have two roos दो मूल $R_1=PA$ & $R_2=PB$

$PA+PB=2/3$; $PA.PB=\frac{4(2+\sqrt{3})}{3} \quad \Rightarrow \quad \frac{1}{PA}+\frac{1}{PB}=\frac{2}{4(2+\sqrt{3})}=\frac{2-\sqrt{3}}{2}$

$PA^2+PB^2=(PA+PB)^2-2PA.PB=\frac{4}{9}+\frac{8(2+\sqrt{3})}{3}=\frac{4+48+24\sqrt{3}}{9}=\frac{2(27+12\sqrt{3})}{9}$

17. Let $u \equiv ax + by + a\sqrt[3]{b} = 0$, $v \equiv bx - ay + b\sqrt[3]{b} = 0$, where $a, b \in R$ be two straight lines, then find the equations of the bisectors of the angles formed by $k_1u - k_2v = 0$ & $k_1u + k_2v = 0$ for non zero real k_1 & k_2 are :

माना दो रेखाएँ $u \equiv ax + by + a\sqrt[3]{b} = 0$ एवं $v \equiv bx - ay + b\sqrt[3]{b} = 0$, जहाँ $a, b \in R$ है। अशून्य वास्तविक संख्याओं k_1 एवं k_2 के लिए $k_1u - k_2v = 0$ एवं $k_1u + k_2v = 0$ से बने कोणों के अर्धकों के समीकरण है –

(A*) $u = 0$ (B) $k_2u + k_1v = 0$ (C) $k_2u - k_1v = 0$ (D*) $v = 0$

Sol. $k_1u - k_2v = 0$ .... (i)

$k_1u + k_2v = 0$ .... (ii)

$\because$ equations of bisectors of the angles formed by lines (i) and (ii) are

$$\frac{k_1u - k_2v}{\sqrt{(ak_1 - bk_2)^2 + (k_1b + ak_2)^2}} = \frac{\pm(k_1u + k_2v)}{\sqrt{(k_1a + bk_2)^2 + (k_1b - ak_2)^2}} \Rightarrow k_1u - k_2v = \pm(k_1u + k_2v) \quad \text{.... (iii)}$$

(i) by taking positive sign in (iii), we get $k_1u - k_2v = k_1u + k_2v \Rightarrow 2k_2v = 0 \Rightarrow v = 0$

(ii) by taking negative sing in (iii), we get $u = 0$

Hindi. $k_1u - k_2v = 0$ .... (i) $k_1u + k_2v = 0$ .... (ii)

$\because$ रेखा (i) व (ii) के कोर्णाधकों की समीकरणें

$$\frac{k_1u - k_2v}{\sqrt{(ak_1 - bk_2)^2 + (k_1b + ak_2)^2}} = \frac{\pm(k_1u + k_2v)}{\sqrt{(k_1a + bk_2)^2 + (k_1b - ak_2)^2}} \Rightarrow k_1u - k_2v = \pm(k_1u + k_2v) \quad \text{.... (iii)}$$

(i) समीकरण (iii) में + ve चिन्ह् लेने पर हम पाते हैं

$k_1u - k_2v = k_1u + k_2v \Rightarrow 2k_2v = 0 \Rightarrow v = 0$

(ii) समीकरण (iii) में – ve चिन्ह् लेने पर हम पाते हैं। $u = 0$

18. The sides of a triangle are the straight line $x + y = 1$, $7y = x$ and $\sqrt{3}y + x = 0$. Then which of the following is an interior points of triangle ?

(A) circumcentre (B*) centroid (C*) incentre (D) orthocentre

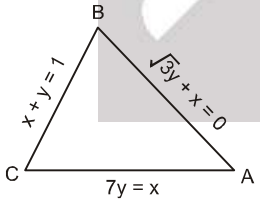
एक त्रिभुज की भुजाओं की समीकरणें $x + y = 1$, $7y = x$ तथा $\sqrt{3}y + x = 0$ है, तो निम्न में से कौन सा (कौन–से) बिन्दु त्रिभुज के अन्दर स्थित है –

(A) परिकेन्द्र (B*) केन्द्रक (C*) अन्तःकेन्द्र (D) लम्बकेंद्र

Sol. Let slope of given lines $m_1 = \frac{1}{7}$, $m_2 = \frac{-1}{\sqrt{3}}$, $m_3 = -1$

Hence interior angle of triangle

$$\tan A = \frac{m_1 - m_2}{1 + m_1m_2} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3} + 7}{7\sqrt{3} - 1} > 0$$



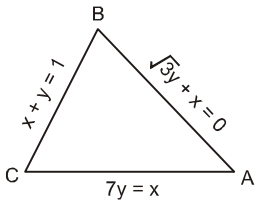
$$\tan B = \frac{m_2 - m_3}{1 + m_2m_3} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} > 0 \quad \Rightarrow \quad \tan C = \frac{m_3 - m_1}{1 + m_2m_1} = \frac{-1 - \frac{1}{7}}{1 - \frac{1}{7}} = \frac{-8}{6} < 0$$

Hence angle C is obtuse therefore circumcentre and orthocentre lies outside the triangle.

Sol. दी गई रेखाओं की प्रवणताऐं क्रमशः है $m_1 = \frac{1}{7}$, $m_2 = \frac{-1}{\sqrt{3}}$, $m_3 = -1$

अतः त्रिभुज के अन्तःकोणों के लिये

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3} + 7}{7\sqrt{3} - 1} > 0$$



$$\tan B = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-\frac{1}{\sqrt{3}} + 1}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} > 0 \quad \Rightarrow \quad \tan C = \frac{m_3 - m_1}{1 + m_2 m_1} = \frac{-1 - \frac{1}{7}}{1 - \frac{1}{7}} = \frac{-8}{6} < 0$$

अतः कोण C अधिक कोण है अतः परिकेन्द्र एवं लम्बकेन्द्र त्रिभुज के बाहर होगें।

19. The line 'ℓ_1' passing through the point (1, 1) and the 'ℓ_2' passes through the point (– 1, 1). If the difference of the slope of lines is 2. Find the locus of the point of intersection of the ℓ_1 and ℓ_2.
रेखा 'ℓ_1', बिन्दु (1, 1) से गुजरती है तथा रेखा 'ℓ_2' बिन्दु (– 1, 1) से गुजरती है यदि रेखाओं की प्रवणताओं का अन्तर 2 है तब ℓ_1 और ℓ_2 के प्रतिच्छेद बिन्दु का बिन्दुपथ ज्ञात कीजिए।

(A*) $x^2 = y$ (B*) $y = 2 - x^2$ (C) $y^2 = x$ (D) $x = 2 - y^2$

Sol. $m_1 - m_2 = 2$ or या $m_1 - m_2 = -2$

$\frac{k-1}{h-1} - \frac{k-1}{h+1} = 2$ $\frac{k-1}{h-1} - \frac{k-1}{h+1} = -2$

....

$x^2 = y$ $y = 2 - x^2$.

20. The two lines pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon, then the correct statement among the following is/are

(A*) Area of polygon is 6 (B*) Length of its diagonals are $\sqrt{5}$ & $\sqrt{53}$

(C) Point of intersection of diagonals is (– 2, 2) (D*) Polygon is parallelogram.

दो रेखाओ के युग्म $y^2 - 4y + 3 = 0$ और $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$, चार भुजाओं के उत्तल बहुभुज का निर्माण करते है तब निम्न में से कौनसा कथन सही है।

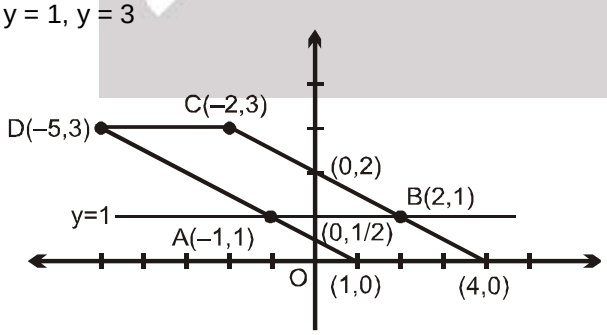
(A*) बहुभुज का क्षेत्रफल 6 है। (B*) इसके विकर्णो की लम्बाई $\sqrt{5}$ और $\sqrt{53}$ है।

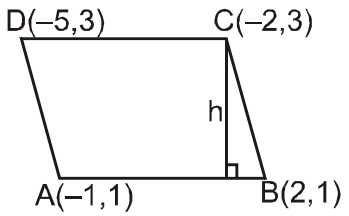
(C) विकर्णो के प्रतिच्छेद बिन्दु (– 2, 2) है। (D*) बहुभुज समान्तर चतुर्भुज है।

Sol. $y^2 - 4y + 3 = 0$ and और $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$(y - 3)(y - 1)$ $(x + 2y - 1)(x + 2y - 4) = 0$

$y = 1, y = 3$





ℓ (AB) = 3 and और h = 2

Area of parallelogram समान्तर चतुर्भुज का क्षेत्रफल= 3 × 2 = 6

$\therefore$ AC = $\sqrt{1^2 + 2^2} = \sqrt{5}$, BD = $\sqrt{7^2 + 2^2} = \sqrt{53}$

21. If the distance between the lines represented $9x^2 - 24xy + 16y^2 + k(6x - 8y) = 0$ is 4, then k may be
यदि $9x^2 - 24xy + 16y^2 + k(6x - 8y) = 0$ से निरूपित रेखाओं के मध्य दुरी 4 है तब k हो सकता है–
(A) 3 (B*) 10 (C*) –10 (D) 7

Sol. Let the lines are $3x - 4y + C_1 = 0$ and $3y - 4y + C_2 = 0$

so according to question $\frac{C_1 - C_2}{\sqrt{16+9}} = 4 \Rightarrow |C_1 - C_2| = 20$

lines are $(3x - 4y + C_1)(3x - 4y + C_2) = 0$ but constant term is zero so C_2 should = 0

$\Rightarrow 9x^2 - 24xy + 16y^2 + C_1(3x - 4y) = 0 \Rightarrow 9x^2 - 24xy + 16y^2 + \frac{C_1}{2}(6x - 4y) = 0$

$\Rightarrow \frac{C_1}{2} = K$ and $C_1 = 20$ so $K = \pm 10$.

Hindi. माना रेखाएं $3x - 4y + C_1 = 0$ और $3y - 4y + C_2 = 0$

इसलिए प्रश्न के अनुसार $\frac{C_1 - C_2}{\sqrt{16+9}} = 4 \Rightarrow |C_1 - C_2| = 20$

रेखाएं $(3x - 4y + C_1)(3x - 4y + C_2) = 0$ परन्तु अचर पद शून्य है इसलिए $C_2 = 0$ होना चाहिए

$\Rightarrow 9x^2 - 24xy + 16y^2 + C_1(3x - 4y) = 0 \Rightarrow 9x^2 - 24xy + 16y^2 + \frac{C_1}{2}(6x - 4y) = 0$

$\Rightarrow \frac{C_1}{2} = K$ और $C_1 = 20$ इसलिए$K = \pm 10$.

PART - IV : COMPREHENSION

भाग - IV : अनुच्छेद (COMPREHENSION)

Comprehenssion # 1 (Q. NO. 1 to 3)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of Δ ABC is G(3, 2), then answer the following questions :

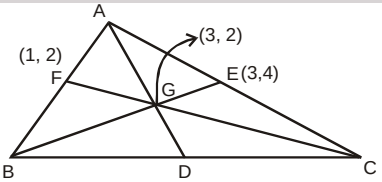
1. The equation of side AB is
(A*) $2x + y = 4$ (B) $x + y - 3 = 0$ (C) $4x - 2y = 0$ (D) none of these

2. Co-ordinates of D are
(A) (7, –4) (B*) (5, 0) (C) (7, 4) (D) (– 3, 0)

3. Height of altitude drawn from point A is (in units)
(A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C*) $6\sqrt{2}$ (D) $2\sqrt{3}$

Sol. **(1, 2, 3)**

Let the co-ordinates of D(α, β) then $\frac{\alpha + 1 + 3}{3} = 3 \quad \Rightarrow \quad \alpha = 5$



and $\frac{\beta + 2 + 4}{3} = 2 \quad \Rightarrow \quad \beta = 0$

$\therefore$ D(5, 0)

Taking $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

then by $\frac{x_1 + x_2}{2} = 1$, $\frac{x_2 + x_3}{2} = 5$, $\frac{x_3 + x_1}{2} = 3$ and $\frac{y_1 + y_2}{2} = 2$, $\frac{y_2 + y_3}{2} = 0$, $\frac{y_1 + y_3}{2} = 4$

we get A(–1, 6) , B(3, –2) , C(7, 2) equation of AB is 2x + y = 4

Height of altitude from A is = $\frac{2\times \text{area}(\Delta ABC)}{BC} = 6\sqrt{2}$

अनुच्छेद # 1 (Q. No. 1 to 3)

मानाकि ABC एक न्यूनकोण त्रिभुज है तथा AD, BE तथा CF इसकी माध्यिकाएँ है जहाँ E तथा F क्रमशः बिन्दु (3, 4) और (1, 2) है तथा त्रिभुज ABC का केन्द्रक G(3, 2) है, तो निम्न प्रश्नों के उत्तर दीजिए।

1. भुजा AB का समीकरण है –

(A*) 2x + y = 4 (B) x + y – 3 = 0 (C) 4x – 2y = 0 (D) इनमें से कोई नहीं

2. D के निर्देशांक है –

(A) (7, –4) (B*) (5, 0) (C) (7, 4) (D) (– 3, 0)

3. बिन्दु A से डाले गये शीर्षाभिलम्ब की ऊँचाई (इकाई में) है –

(A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C*) $6\sqrt{2}$ (D) $2\sqrt{3}$

Sol. (A, B, C)

मानाकि D के निर्देशांक (α, β) है तो $\frac{\alpha+1+3}{3} = 3 \Rightarrow \alpha = 5$ तथा $\frac{\beta+2+4}{3} = 2 \Rightarrow \beta = 0$

$\therefore$ D(5, 0)

$A(x_1, y_1)$, $B(x_2, y_2)$ और $C(x_3, y_3)$ लेने पर

$\frac{x_1+x_2}{2} = 1, \frac{x_2+x_3}{2} = 5, \frac{x_3+x_1}{2} = 3$

तथा $\frac{y_1+y_2}{2} = 2, \frac{y_2+y_3}{2} = 0, \frac{y_1+y_3}{2} = 4$

$\Rightarrow$ A(–1, 6) , B(3, –2) , C(7, 2)

AB का समीकरण है 2x + y = 4 ; A से शीर्षाभिलम्ब की ऊँचाई $\frac{2\times(\Delta ABC \text{ का क्षे०})}{BC} = 6\sqrt{2}$

Comprehension # 2 (Q.No. 4 to 6)

If vertices of triangle are $P(p_1, p_2)$, $Q(q_1, q_2)$, $R(r_1, r_2)$, then area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ and if P, Q, R are collinear, then $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0$.

On the basis of above answer the following question.

4. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of median through A.

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B*) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) None of these

5. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of line through A and parallel to BC

(A*) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$

6. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find the equation of internal angle bisector through A

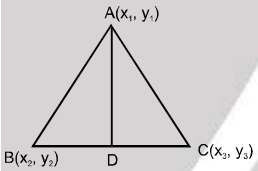
(A) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (B) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C*) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (D) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Sol. (A) D is mid point of BC. Hence co-ordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Therefore, equation of the median AD is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix} = 0$$



Applying $R_3 \to 2R_3$

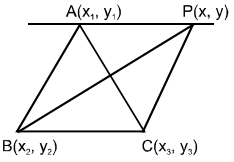
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2+x_3 & y_2+y_3 & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(using the addition property of determinats)

(B) Let P(x, y) be any point on the line parallel to BC

Area of ΔABP = Area of ΔACP

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



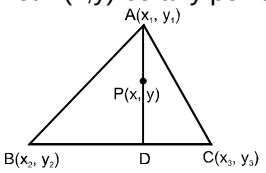
$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

This gives the equation of line AP.

(C) Let AD be the internal bisector of angle A,

$\therefore \frac{BD}{DC} = \frac{BA}{CA} = \frac{c}{b}$ $\quad \therefore D \equiv \left(\frac{cx_3 + bx_2}{c+b}, \frac{cy_3 + by_2}{c+b}\right)$

Let P(x,y) be any point on AD then P,A,D are collinear



$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{cx_3 + bx_2}{b+c} & \frac{cy_3 + by_2}{b+c} & 1 \end{vmatrix} = 0$$

$R_3 \to (b + c) R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 + bx_2 & cy_3 + by_2 & b+c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 & cy_3 & c \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ bx_2 & by_2 & b \end{vmatrix} = 0 \qquad \text{(Addition property)}$$

$$\Rightarrow c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

This is the equation of AD.

(D)

Hindi अनुच्छेद # 2 (Q.No. 4 to 6)

यदि त्रिभुज के शीर्ष $P(p_1, p_2)$, $Q(q_1, q_2)$, $R(r_1, r_2)$ तब ΔPQR का क्षेत्रफल $= \frac{1}{2}\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ और P, Q, R संरेख है तब $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0$. इस आधार पर निम्न प्रश्नों का उत्तर दीजिए

4. यदि $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ त्रिभुज के शीर्ष है तब A से जाने वाली माध्यिका का समीकरण है–.

(A) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(B*) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$

(D) इनमें से कोई नहीं।

5. यदि $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ त्रिभुज के तीन शीर्ष है तब A से जाने वाली BC के समान्तर रेखा का समीकरण है–

(A*) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$ (B) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C) $\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$ (D) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$

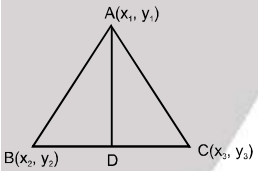
6. यदि $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ त्रिभुज के शीर्ष है तब A से जाने वाले आन्तरीक कोण अर्द्धक का समीकरण है–

(A) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (B) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(C*) $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ (D) $c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Sol. (A) D, BC का मध्य बिन्दु है। अतः D के निर्देशांक $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

अतः माध्यिका AD का समीकरण है $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix} = 0$



$R_3 \to 2R_3$

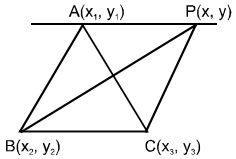
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2+x_3 & y_2+y_3 & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(सारणिक के योगफल प्रगुण का प्रयोग करने पर)

(B) माना P(x, y), BC के समान्तर रेखा पर एक बिन्दु है

ΔABP का क्षेत्रफल = ΔACP का क्षेत्रफल

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



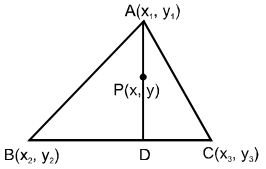
$$\Rightarrow \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

यह रेखा AP का समीकरण है।

(C) माना AD कोण A का आन्तरिक समद्विभाजक है।

$\therefore \frac{BD}{DC} = \frac{BA}{CA} = \frac{c}{b}$ $\quad \therefore D \equiv \left(\frac{cx_3 + bx_2}{c+b}, \frac{cy_3 + by_2}{c+b}\right)$

माना P(x,y), AD पर कोई बिन्दु है, तो P,A,D संरेख है।



$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{cx_3 + bx_2}{b+c} & \frac{cy_3 + by_2}{b+c} & 1 \end{vmatrix} = 0$$

$R_3 \to (b + c) R_3$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 + bx_2 & cy_3 + by_2 & b+c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ cx_3 & cy_3 & c \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ bx_2 & by_2 & b \end{vmatrix} = 0$$ (योगफल प्रगुण)

$$\Rightarrow c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

यह AD का समीकरण है।

(D)

COMPREHENSION#3 (7-9)

The equation of pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$

The equation of pair of straight lines bisecting the angles between the straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{(x-\alpha)^2 - (y-\beta)^2}{a-b} = \frac{(x-\alpha)(y-\beta)}{h}$ where (α, β) is point of intersecting of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

7. Find the combined equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$

(A*) $5x^2 - 2xy - 5y^2 = 0$ (B) $5x^2 - 5y^2 + 4xy = 0$

(C) $5x^2 - 5y^2 + 2xy = 0$ (D) $5x^2 - 5y^2 - xy = 0$

8. Find the combined equation of the bisectors of the angle between the lines represented by $2x^2 - 3y^2 + xy - x + 11y - 10 = 0$

(A) $x^2 - y^2 + xy + 4x + 2 = 0$

(B) $x^2 - y^2 - 10xy + 8x + 12y - 10 = 0$

(C) $x^2 - y^2 - 5xy + 3x + 7y - 5 = 0$
(D*) None of these

Sol. Point of intersection of lines $2x^2 - 3y^2 + xy - x + 11y - 10 = 0$ is $\left(-\frac{1}{5}, \frac{9}{5}\right)$

equation of pair of angle bisector is $\frac{(x+1/5)^2 - (y-9/5)^2}{2+3} = \frac{(x+1/5)(y-9/5)}{1/2}$

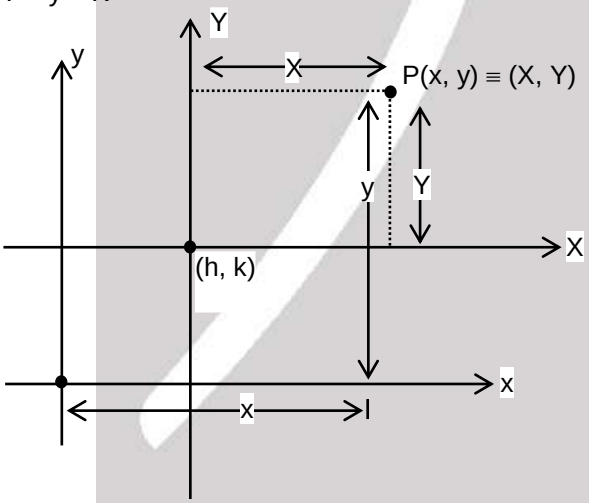
9. Let combined equation of lines ℓ_1 and ℓ_2 is $x^2 - 2pxy - y^2 = 0$ combined equation of lines ℓ_3 and ℓ_4 is $x^2 - 2qxy - y^2 + 2gx + 2fy + c = 0$. If angle between lines (ℓ_1, ℓ_3), (ℓ_1, ℓ_4), (ℓ_2, ℓ_3) and (ℓ_2, ℓ_4) is equal to 45°, then find the value of pq.
(A) –4 (B) 4 (C*) –1 (D) 1

Sol. $x^2 - 2pxy - y^2 = 0$ is same as $\frac{x^2 - y^2}{1-(-1)} = \frac{xy}{-p}$ (angle bisector of $x^2 - 2pxy - y^2 = 0$)

$\Rightarrow \quad 2q = \frac{2}{-p} \quad \Rightarrow \quad pq = -1$

Comprehension (10 & 11)

Origin of coordinate system xy is shifted to (h, k) to make new coordinate system XY. X and Y are parallel to x and y. New co-ordinates of point P(x, y) are P(X, Y). x, y, X, Y are related as given below.
xy निर्देशांक पद्धति के मूल बिन्दु को (h, k) से स्थानान्तरित किया जाता है तथा नया निर्देशांक पद्धति XY बनायी जाती है। X और Y, x और y के समान्तर बिन्दु है P(x, y) के नये निर्देशांक P(X, Y) है। x, y, X, Y निम्न तरह से सम्बन्धित है
$X = x - h$
$Y = y - K$



10. Co-ordinates of (–7, 9) if origin is shifted to (2, 4) without changing direction of axes, are
(–7, 9) के निर्देशांक है यदि अक्षों की दिशा को परिवर्तित किए बिना मूल बिन्दु को (2, 4) से स्थानान्तरित किया जाता है।
(A) (–5,13) (B) (–7,4) (C*) (–9,5) (D) (–9,13)

Sol. $x = -7, y = 9$
$h = 2, k = 4$
$(X, Y) = (x - h, y - k) = (-9, 5)$

11. If co-ordinate axes are so translated such that ordinate of (4, 12) becomes zero while abscissa remains same. Then new coordinates of point (–8, –2) are
यदि निर्देशांक अक्षों को इस प्रकार रूपान्तरित किया जाता है कि (4, 12) की कोटि शून्य हो जाती है जबकि भुज समान रहता है। बिन्दु (–8, –2) के नये निर्देशांक है –
(A) (–8, 14) (B) (–8, 10) (C*) (–8, –14) (D) (–8, –10)

Sol. $x = 4, y = 12$

(X, Y) = (4 – h, 12 – k) = (4 – h, 0)
k = 12, h = 0
(X, Y) = (–8 – 0, –2 – 12) = (–8, –14)

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

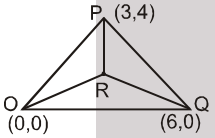
भाग - I : JEE (ADVANCED) / IIT-JEE (पिछले वर्षो) के प्रश्न

*** Marked Questions may have more than one correct option.**
*** चिन्हित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है -**

1. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are
[IIT-JEE - 2007, P-II, (3, – 1), 81]
माना कि O(0, 0), P(3, 4), Q(6, 0) त्रिभुज OPQ के शीर्षबिन्दु (vertices) हैं। R त्रिभुज OPQ का अंदर एक ऐसा बिन्दु है कि त्रिभुजों OPR, PQR, OQR के क्षेत्रफल बराबर है। R के निर्देशांक (co-ordinates) हैं –
[IIT-JEE - 2007, P-II, (3, – 1), 81]

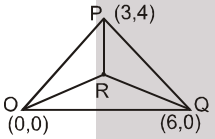
(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C*) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

Sol.



R is centroid hence R ≡ $\left(3, \frac{4}{3}\right)$

Hindi.



R केन्द्रक है अतः R ≡ $\left(3, \frac{4}{3}\right)$

2. Lines $L_1 : y – x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[IIT-JEE - 2007, P-II, (3, – 1), 81]**

STATEMENT – 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.
because
STATEMENT – 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement - 1 is True, Statement - 2 is True; Statement-2 **is** a correct explanation for Statement - 1
(B) Statement -1 is True, Statement - 2 is True; Statement-2 is **NOT** a correct explanation for Statement - 1
(C*) Statement - 1 is True, Statement - 2 is False
(D) Statement - 1 is False, Statement - 2 is True

रेखायें $L_1 : y – x = 0$ तथा $L_2 : 2x + y = 0$ रेखा $L_3 : y + 2 = 0$ को क्रमशः P तथा Q पर काटती है। L_1 तथा L_2 के बीच के न्यूनकोण (acute angle) का समद्विभाजक (bisector) L_3 को R पर काटता है।

वक्तव्य - 1 : PR : RQ का अनुपात $2\sqrt{2} : \sqrt{5}$ है। **[IIT-JEE - 2007, P-II, (3, – 1), 81]**
क्योंकि

वक्तव्य - 2 : किसी भी त्रिभुज के एक कोण का समद्विभाजक त्रिभुज को दो समरूप (similar) त्रिभुजों में विभाजित करता है।

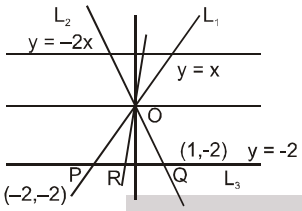
(A) वक्तव्य-1 सत्य है, वक्तव्य-2 सत्य है; वक्तव्य-2 वक्तव्य-1 का सही स्पष्टीकरण हैं।

(B) वक्तव्य-1 सत्य है, वक्तव्य-2 सत्य है; वक्तव्य-2 वक्तव्य-1 का सही स्पष्टीकरण नहीं हैं।

(C) वक्तव्य-1 सत्य है, वक्तव्य-2 असत्य है।

(D) वक्तव्य-1 असत्य है, वक्तव्य-2 सत्य है।

Sol. $\frac{PR}{RQ} = \frac{OP}{OQ}$

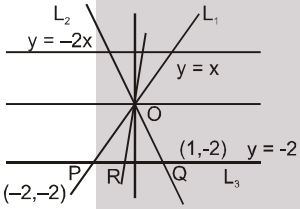


$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$

but statement – 2 is false

$\therefore$ Ans. (C)

Hindi. $\frac{PR}{RQ} = \frac{OP}{OQ}$



$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$

परन्तु कथन – 2 असत्य है। $\therefore$ **Ans. (C)**

3. Consider three points **[IIT-JEE - 2008, P-II, (3, – 1), 81]**

$P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$ and $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, where

$0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment QP (D*) P, Q, R are non-collinear

तीन बिन्दु लीजिये $P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$ तथा $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$,

जहाँ $0 < \alpha, \beta, \theta < \frac{\pi}{4}$ है। तो **[IIT-JEE - 2008, P-II, (3, – 1), 81]**

(A) P रेखाखण्ड RQ पर है (B) Q रेखाखण्ड PR पर है

(C) R रेखाखण्ड QP पर है (D) P, Q, R असंरेख हैं

Sol. $P \equiv (-\sin(\beta-\alpha), -\cos\beta)$; $Q \equiv (\cos(\beta-\alpha), \sin\beta)$

$R \equiv (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$ $0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$x_R = \cos(\beta-\alpha)\cos\theta - \sin(\beta-\alpha)\sin\theta \Rightarrow x_R = x_Q \cdot \cos\theta + x_P \cdot \sin\theta$

$y_R = \sin\beta\cos\theta - \cos\beta\sin\theta \Rightarrow y_R = y_Q \cdot \cos\theta + y_P \cdot \sin\theta$

For P, Q, R to be collinear $\sin\theta + \cos\theta = 1$

$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $\Rightarrow$ not possible for the given interval $\theta \in \left(0, \frac{\pi}{4}\right)$ $\Rightarrow$ non collinear

Hindi : $P \equiv (-\sin(\beta-\alpha), -\cos\beta)$; $Q \equiv (\cos(\beta-\alpha), \sin\beta)$

$R \equiv (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$ $\quad 0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$x_R = \cos(\beta-\alpha)\cos\theta - \sin(\beta-\alpha)\sin\theta \Rightarrow x_R = x_Q \cdot \cos\theta + x_P \cdot \sin\theta$

$y_R = \sin\beta\cos\theta - \cos\beta\sin\theta \Rightarrow y_R = y_Q \cdot \cos\theta + y_P \cdot \sin\theta$

P, Q, R के संरेखीय होने के लिए $\sin\theta + \cos\theta = 1$

$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Rightarrow$ जो दिए गए अन्तराल $\theta \in \left(0, \frac{\pi}{4}\right)$ के लिए सम्भव नहीं है।

$\Rightarrow$ असंरेखीय

4. The locus of the orthocentre of the triangle formed by the lines **[IIT-JEE - 2009, Paper-2, (3, –1), 80]**
$(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is
(A) a hyperbola (B) a parabola (C) an ellipse (D*) a straight line

रेखाओं $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ एवं $y = 0$ जहाँ $p \neq q$ से बने त्रिभुज के लम्ब–केन्द्र का बिन्दुपथ है – **[IIT-JEE - 2009, Paper-2, (3, –1), 80]**

(A) अतिपरवलय (B) परवलय (C) दीर्घवृत्त (D) एक सरल रेखा

Sol. **(D)**

$(1 + p)x - py + p(1 + p) = 0$ .....(1)

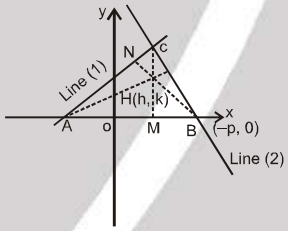
$(1 + q)x - qy + q(1 + q) = 0$ .....(2)

on solving (1) and (2), we get C(pq, (1 + p) (1 + q))

$\therefore$ equation of altitude CM passing through C and perpendicular to AB is $x = pq$ .....(3)

$\because$ slope of line (2) is $= \left(\frac{1+q}{q}\right)$

$\therefore$ slope of altitude BN (as shown in figure) is $= \frac{-q}{1+q}$



$\therefore$ equation of BN is $y - 0 = \frac{-q}{1+q}(x + p)$

$\Rightarrow y = \frac{-q}{(1+q)}(x + p)$ ..... (4)

Let orthocentre of triangle be H(h, k) which is the point of intersection of (3) and (4)

$\therefore$ on solving (3) and (4), we get

$x = pq$ and $y = -pq \Rightarrow h = pq$ and $k = -pq$

$\therefore h + k = 0$

$\therefore$ locus of H(h, k) is $x + y = 0$

Hindi $(1 + p)x - py + p(1 + p) = 0$ .....(1)

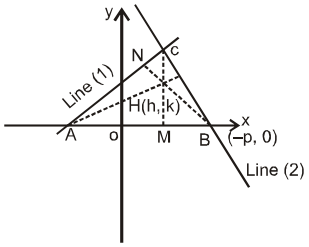
$(1 + q)x - qy + q(1 + q) = 0$ .....(2)

(1) तथा (2) को हल करने पर C(pq, (1 + p) (1 + q))

$\therefore$ शीर्षलम्ब CM का समीकरण जो C से गुजरता है तथा AB के लम्बवत है $x = pq$ .....(3)

$\because$ रेखा (2) की प्रवणता $= \left(\frac{1+q}{q}\right)$

$\therefore$ शीर्षलम्ब BN की प्रवणता (चित्रानुसार) $= \frac{-q}{1+q}$



$\therefore$ BN का समीकरण $y - 0 = \frac{-q}{1+q}(x + p)$

$\Rightarrow$ $y = \frac{-q}{(1+q)}(x + p)$ ..... (4)

मानाकि त्रिभुज का लम्बकेन्द्र H(h, k) है, जोकि (3) तथा (4) का प्रतिच्छेद बिन्दु है

$\therefore$ (3) और (4) को हल करने पर

$x = pq$ और $y = -pq$ $\Rightarrow$ $h = pq$ और $k = -pq$

$\therefore$ $h + k = 0$

$\therefore$ अतः लम्बकेन्द्र H(h, k) का बिन्दुपथ $x + y = 0$

5. A straight line **L** through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If **L** also intersects the x-axis, then the equation of **L** is

एक सरल रेखा L बिन्दु (3, –2) से गुजरती है, और रेखा $\sqrt{3}x + y = 1$ से 60º कोण पर झुकी है। यदि L, x- अक्ष को भी प्रतिच्छेद करती है रेखा $y = 0$ को काटती है, तो L का समीकरण है **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B*) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Ans. (B)

Sol. Let slope of line L = m

$\therefore$ $\left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| = \tan 60^\circ = \sqrt{3} \Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

taking positive sign, $m + \sqrt{3} = \sqrt{3} - 3m$ $\Rightarrow$ $m = 0$

taking negative sign $m + \sqrt{3} + \sqrt{3} - 3m = 0$ $\Rightarrow$ $m = \sqrt{3}$

As L cuts x-axis $\Rightarrow$ $m = \sqrt{3}$

so L is $y + 2 = \sqrt{3}(x - 3)$

Hindi माना रेखा L की प्रवणता = m

$\therefore$ $\left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| = \tan 60^\circ = \sqrt{3} \Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

धनात्मक चिन्ह लेने पर $m + \sqrt{3} = \sqrt{3} - 3m$ $\Rightarrow$ $m = 0$

ऋणात्मक चिन्ह लेने पर $m + \sqrt{3} + \sqrt{3} - 3m = 0$ $\Rightarrow$ $m = \sqrt{3}$

चूंकि L, x-अक्ष को प्रतिच्छेद करती है $\Rightarrow$ $m = \sqrt{3}$

इसलिए L का समीकरण $y + 2 = \sqrt{3}(x - 3)$

6. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

$a > b > c > 0$ के लिए (1, 1) तथा रेखाओं $ax + by + c = 0$ व $bx + ay + c = 0$ के प्रतिच्छेद बिन्दु के बीच की दूरी $2\sqrt{2}$ से कम है, तब

(A) $a + b - c > 0$ (B) $a - b + c < 0$ (C) $a - b + c > 0$ (D) $a + b - c < 0$

Sol. (A) or (C) or Bonus

As $a > b > c > 0$ $\Rightarrow$ $a - c > 0$ and $b > 0$

$\Rightarrow$ $a - c > 0$ and $b > 0$

$\Rightarrow$ $a + b - c > 0$ $\Rightarrow$ option (A) is correct

Further $a > b$ and $c > 0$

$\Rightarrow$ $a - b > 0$ and $c > 0$

$\Rightarrow$ $a - b > 0$ and $c > 0$

$\Rightarrow$ $a - b + c > 0$ $\Rightarrow$ option (c) is correct

Aliter

$(a - b)x + (b - a)y = 0 \Rightarrow x = y \Rightarrow$ Point of intersection $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Now $\sqrt{\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2} < 2\sqrt{2} \Rightarrow \sqrt{2}\left(\frac{a+b+c}{a+b}\right) < 2\sqrt{2} \Rightarrow a + b - c > 0$

Hindi **(A)** या **(C)** या **Bonus**

चूंकि $a > b > c > 0$ $\Rightarrow$ $a - c > 0$ तथा $b > 0$

$\Rightarrow$ $a - c > 0$ तथा $b > 0$

$\Rightarrow$ $a + b - c > 0$ $\Rightarrow$ विकल्प (A) सही है।

यद्यपि $a > b$ तथा $c > 0$

$\Rightarrow$ $a - b > 0$ तथा $c > 0$

$\Rightarrow$ $a - b > 0$ तथा $c > 0$

$\Rightarrow$ $a - b + c > 0$ $\Rightarrow$ विकल्प (c) सही है।

Aliter

$(a - b)x + (b - a)y = 0 \Rightarrow x = y \Rightarrow$ प्रतिच्छेद बिन्दु $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

अब $\sqrt{\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2} < 2\sqrt{2} \Rightarrow \sqrt{2}\left(\frac{a+b+c}{a+b}\right) < 2\sqrt{2} \Rightarrow a + b - c > 0$

7. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

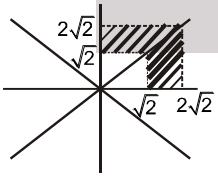
समतल में स्थित किसी बिन्दु P से रेखाओं $x - y = 0$ तथा $x + y = 0$ की दूरी क्रमशः $d_1(P)$ तथा $d_2(P)$ है। यदि क्षेत्र R उन सभी बिन्दुओं P से बना है जो प्रथम चतुर्थांश (quadrant) में स्थित है तथा $2 \le d_1(P) + d_2(P) \le 4$ को संतुष्ट करते है, तब क्षेत्र R का क्षेत्रफल है।

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

Ans. **(6)**

Sol. let p(h, k)

$$2 \le \left|\frac{h-k}{\sqrt{2}}\right| + \left|\frac{h+k}{\sqrt{2}}\right| \le 4$$



$\Rightarrow$ $2\sqrt{2} \le |h - k| + |h + k| \le 4\sqrt{2}$

if $h \ge k$

$\Rightarrow$ $2\sqrt{2} \le x - y + x + y \le 4\sqrt{2}$ or $\sqrt{2} \le x \le 2\sqrt{2}$

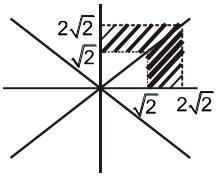
similarly when $k > h$

we have $\sqrt{2} \le y \le 2\sqrt{2}$

The required area = $(2\sqrt{2})^2 - (\sqrt{2})^2 = 6$.

Hindi. माना p(h, k)

$2 \le \left|\frac{h-k}{\sqrt{2}}\right| + \left|\frac{h+k}{\sqrt{2}}\right| \le 4$



$\Rightarrow\ 2\sqrt{2} \le |h-k| + |h+k| \le 4\sqrt{2}$

यदि $h \ge k$

$\Rightarrow\ 2\sqrt{2} \le x - y + x + y \le 4\sqrt{2}$ या $\sqrt{2} \le x \le 2\sqrt{2}$

इसप्रकार $k > h$

यहाँ $\sqrt{2} \le y \le 2\sqrt{2}$

अभीष्ट क्षेत्रफल $= (2\sqrt{2})^2 - (\sqrt{2})^2 = 6.$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

भाग - II : JEE (MAIN) / AIEEE (पिछले वर्षो) के प्रश्न

1. The line L given by $\frac{x}{5}+\frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3} = 1$. Then the distance between L and K is **[AIEEE - 2010 (8, –2), 144]**

$\frac{x}{5}+\frac{y}{b} = 1$ द्वारा निरूपित रेखा L, बिन्दु (13, 32) से होकर जाती है। रेखा K, रेखा L के समान्तर है तथा उसका समीकरण $\frac{x}{c}+\frac{y}{3} = 1$ है। तो L तथा K के बीच की दूरी है– **[AIEEE - 2010 (8, –2), 144]**

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3*) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

Ans. **(3)**

Sol. $\frac{x}{5}+\frac{y}{b} = 1$

$\frac{13}{5}+\frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5} \Rightarrow b = -20$

$\frac{x}{5}-\frac{y}{20} = 1 \Rightarrow 4x - y = 20$

Line K has same slope $\Rightarrow -\frac{3}{c} = 4$

रेखा K की समान प्रवणता है $\Rightarrow -\frac{3}{c} = 4$

$c = -\frac{3}{4} \Rightarrow 4x - y = -3$

distance दूरी $= \frac{23}{\sqrt{17}}$

Hence correct option is (3) अतः सही विकल्प (3) है।

2. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE - 2011, I(4, –1), 120]**

Statement-1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true ; Statement-2 is **not** a correct explanation for Statement-1

(3*) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

रेखा $L_1 : y - x = 0$ तथा $L_2 : 2x + y = 0$, रेखा $L_3 : y + 2 = 0$ को क्रमशः P तथा Q पर प्रतिच्छेद करते हैं। L_1 तथा L_2 के बीच का न्यून कोण समद्विभाजक L_3 को R पर काटता है। **[AIEEE - 2011, I(4, –1), 120]**

कथन-1 : अनुपात PR : RQ, $2\sqrt{2} : \sqrt{5}$ के बराबर है।

कथन-2 : किसी भी त्रिभुज में, एक कोण का समद्विभाजक त्रिभुज को दो समरूप त्रिभुजों में बांटता है।

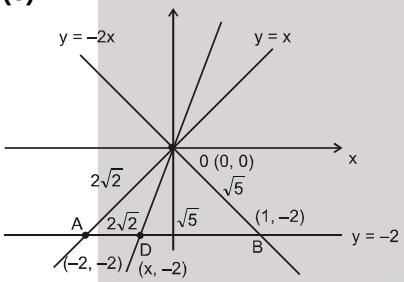
(1) कथन-1 सत्य है, कथन-2 सत्य है। कथन-2, कथन-1 की सही व्याख्या है।

(2) कथन-1 सत्य है, कथन-2 सत्य है। कथन-2, कथन-1 की सही व्याख्या **नहीं** है।

(3*) कथन-1 सत्य है, कथन-2 असत्य है।

(4) कथन-1 असत्य है, कथन-2 सत्य है।

Sol. **(3)**



$$\frac{AD}{DB} = \frac{OA}{OB}$$

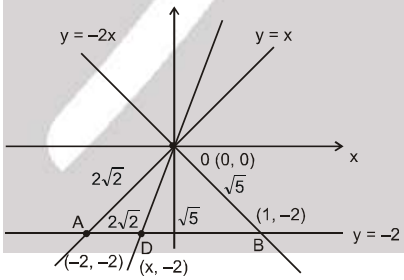
$\therefore$ AD : DB = $2\sqrt{2} : \sqrt{5}$

$\because$ OD is angle bisector of angle AOB

$\therefore$ St : 1 true

St. 2 false (obvious) **Ans.**

Hindi.



$$\frac{AD}{DB} = \frac{OA}{OB}$$

$\therefore$ AD : DB = $2\sqrt{2} : \sqrt{5}$

$\because$ OD कोण AOB का कोण अर्द्धक है

$\therefore$ St : 1 सत्य है

St. 2 असत्य है (स्पष्ट) **Ans.**

3. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE - 2011, II(4, –1), 120]**

रेखाऐं $x + y = |a|$ तथा $ax - y = 1$ प्रथम चतुर्थांश में प्रतिच्छेद करती हैं तो a के सभी संभावित मानों का अन्तराल है : **[AIEEE - 2011, II(4, –1), 120]**

(1) $(0, \infty)$ (2*) $[1, \infty)$ (3) $(-1, \infty)$ (4) $(-1, 1]$

Sol. $x + y = |a| \quad \Rightarrow \quad ax - y = 1$

If यदि $a > 0$

$x + y = a$

$ax - y = 1$

$x(1 + a) = 1 + a$ as $x = 1$

$y = a - 1$

It is in the first quadrant यह प्रथम चतुर्थांश में है।

so अतः $a - 1 \geq 0 \quad \Rightarrow \quad a \geq 1 \quad \Rightarrow \quad a \in [1, \infty)$

If यदि $a < 0$

$x + y = -a$

$ax - y = 1$

+

$x(1 + a) = 1 - a$

$x = \dfrac{1-a}{1+a} > 0 \Rightarrow \dfrac{a-1}{a+1} < 0 \Rightarrow a \in (-1, 1)$ .....(1)

$y = -a - \dfrac{1-a}{1+a} = \dfrac{-a-a^2-1+a}{1+a} > 0$

$-\left(\dfrac{a^2+1}{a+1}\right) > 0 \Rightarrow \dfrac{a^2+1}{a+1} < 0 \Rightarrow a \in (-\infty, -1)$ .....(2)

from (1) and (2) $a \in \{\phi\}$

अतः (1) तथा (2) से $a \in \{\phi\}$

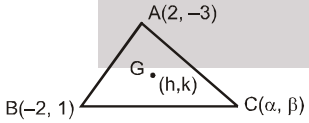
इसलिए सही उत्तर $a \in [1, \infty)$

4. If A(2, –3) and B(–2, 1) are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[AIEEE - 2011, II(4, –1), 120]**

यदि एक त्रिभुज के दो शीर्ष A(2, –3) तथा B(–2, 1) है तथा तीसरा शीर्ष रेखा $2x + 3y = 9$ पर गति (moves) करता है तो त्रिभुज के केन्द्रक का बिन्दुपथ है :

(1) $x - y = 1$ (2*) $2x + 3y = 1$ (3) $2x + 3y = 3$ (4) $2x - 3y = 1$

Sol. $\dfrac{\alpha+2-2}{3} = h \quad \dfrac{-3+1+\beta}{3} = k$



$\alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

third vertex on the line $2x + 3y = 9$ तीसरा शीर्ष सरल रेखा $2x + 3y = 9$ पर है।

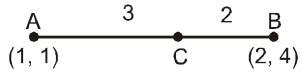
$2\alpha + 3\beta = 9 \quad \Rightarrow \quad 2(3h) + 3(3k + 2) = 9 \quad \Rightarrow \quad 2h + 3k = 1 \quad \Rightarrow \quad 2x + 3y - 1 = 0$

5. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE-2012, (4, –1)/120]**

यदि रेखा 2x + y = k उस बिंदु से होकर जाती है, जो बिंदुओं (1, 1) तथा (2, 4) को मिलाने वाले रेखाखण्ड को 3 : 2 के अनुपात में बांटता है, तो k का मान है–

(1) $\frac{29}{5}$ (2) 5 (3*) 6 (4) $\frac{11}{5}$

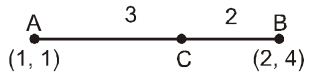
Sol. $\therefore C\left(\frac{8}{5}, \frac{14}{5}\right)$



Line 2x + y = k passes $C\left(\frac{8}{5}, \frac{14}{5}\right) \Rightarrow \frac{2\times 8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

Hindi Ans. (3)

$\therefore C\left(\frac{8}{5}, \frac{14}{5}\right)$



सरल रेखा 2x + y = k बिन्दु $C\left(\frac{8}{5}, \frac{14}{5}\right)$ से गुजरती है।

$\frac{2\times 8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

6. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE-2012, (4, –1)/120]**

बिंदु (1, 2) से होकर जाती एक रेखा इस प्रकार खींची गई है कि वह निर्देशांक अक्षों को P तथा Q पर प्रतिच्छेद कर त्रिभुज OPQ बनाती है, जहाँ O मूल बिंदु है। यदि त्रिभुज OPQ का क्षेत्रफल न्यूनतम है तो रेखा PQ की प्रवणता है :

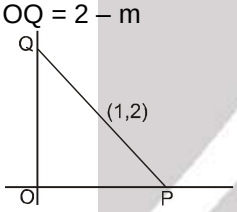
(1) $-\frac{1}{4}$ (2) – 4 (3*) – 2 (4) $-\frac{1}{2}$

Sol. Ans. (3)

(y – 2) = m(x – 1)

OP = $1 - \frac{2}{m}$

OQ = 2 – m



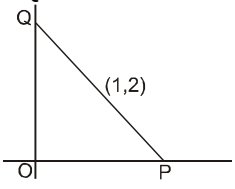
Area of $\Delta POQ = \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(1-\frac{2}{m}\right)(2-m) = \frac{1}{2}\left[2-m-\frac{4}{m}+2\right] = \frac{1}{2}\left[4-\left(m+\frac{4}{m}\right)\right]$

For least area $m = \frac{4}{m}$ & $m < 0 \Rightarrow m = -2$

Hindi. (y – 2) = m(x – 1)

OP = $1 - \frac{2}{m}$

OQ = 2 – m



ΔPOQ का क्षेत्रफल $= \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(1-\frac{2}{m}\right)(2-m) = \frac{1}{2}\left[2-m-\frac{4}{m}+2\right] = \frac{1}{2}\left[4-\left(m+\frac{4}{m}\right)\right]$

न्यूनतम क्षेत्रफल के लिए $m = \frac{4}{m}$ & $m < 0 \Rightarrow m = -2$

7. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

$x + \sqrt{3}y = \sqrt{3}$ की दिशा में जाती हुई एक प्रकाश की किरण x-अक्ष पर पहुँच कर परावर्तित हो जाती है। इस परावर्तित किरण का समीकरण है– **[AIEEE - 2013, (4, –1),360]**

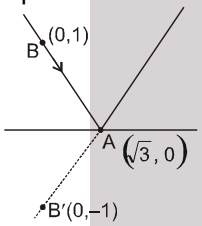
(1) $y = x + \sqrt{3}$ (2*) $\sqrt{3}y = x - \sqrt{3}$ (3) $y = \sqrt{3}x - \sqrt{3}$ (4) $\sqrt{3}y = x - 1$

Sol. **(2)**

Take any point B(0, 1) on given line

B' is image of B w.r.t. x-axis

$\therefore$ Equation of AB′



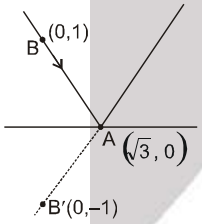
$$y - 0 = \frac{-1-0}{0-\sqrt{3}}\left(x-\sqrt{3}\right)$$

$$-\sqrt{3}y = -x + \sqrt{3}$$

$$x - \sqrt{3}y = \sqrt{3} \quad \Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$$

Hindi. दी गई रेखा पर कोई बिन्दु B(0, 1) लेने पर

x-अक्ष के सापेक्ष B का प्रतिबिम्ब B' है। अतः AB′ का समीकरण



$$y - 0 = \frac{-1-0}{0-\sqrt{3}}\left(x-\sqrt{3}\right)$$

$$-\sqrt{3}y = -x + \sqrt{3}$$

$$x - \sqrt{3}y = \sqrt{3} \quad \Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$$

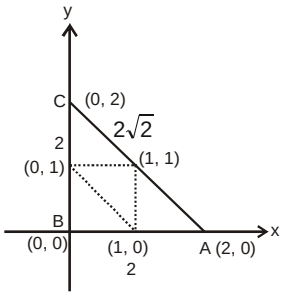
8. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[AIEEE - 2013, (4, –1),360]**

एक त्रिभुज, जिसकी भुजाओं के मध्य बिन्दुओं के निर्देशांक (0, 1) (1, 1) तथा (1, 0) हैं, के अंतः केन्द्र का x-निर्देशांक है :

(1) $2 + \sqrt{2}$ (2*) $2 - \sqrt{2}$ (3) $1 + \sqrt{2}$ (4) $1 - \sqrt{2}$

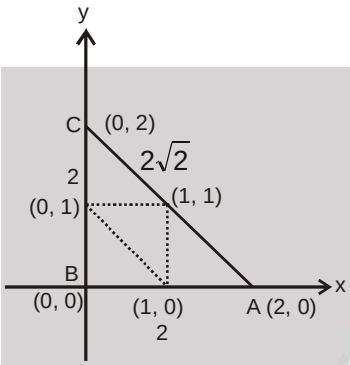
[AIEEE - 2013, (4, –1),360]

Sol.



x - coordinate of incentre = $\frac{2\times 0+2\sqrt{2}.0+2.2}{2+2+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$

Hindi



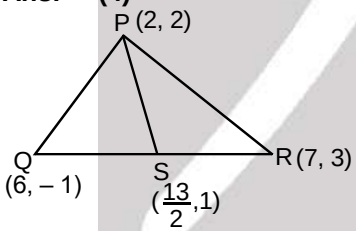
अन्तकेन्द्र का x निर्देशांक = $\frac{2\times 0+2\sqrt{2}.0+2.2}{2+2+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$

9. Let PS be the median of the triangle with vertices P(2, 2), Q (6, – 1), and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is : **[JEE(Main) 2014, (4, – 1), 120]**

माना PS एक त्रिभुज की माध्यिका है जिसके शीर्ष P(2, 2), Q (6, – 1), तथा R (7, 3) है। (1, – 1) से होकर जाने वाली रेखा जो PS के समांतर है, का समीकरण है : **[JEE(Main) 2014, (4, – 1), 120]**

(1) 4x + 7y + 3 = 0 (2) 2x – 9y – 11 = 0 (3) 4x – 7y – 11 = 0 (4) 2x + 9y + 7 = 0

Sol. Ans. (4)



Slope of PS की प्रवणता = $\frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$

Hence equation of line through (1, –1) & parallel to PS is:

अतः (1, –1) से जाने वाली और PS के समान्तर रेखा का समीकरण:

$(y+1) = \frac{-2}{9}(x-1)$

$9y + 2x + 7 = 0$

10. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines 4ax + 2ay + c = 0 and 5bx + 2by + d = 0 lies in the fourth quadrant and is equidistant from the two axes then : **[JEE(Main) 2014, (4, – 1), 120]**

माना a, b, c तथा d शून्येतर संख्याएँ है। यदि रेखाओं 4ax + 2ay + c = 0 तथा 5bx + 2by + d = 0 का प्रतिच्छेद बिंदु चौथे चतुर्थांश में है तथा दोनों अक्षों से समदूरस्थ है, तो : **[JEE(Main) 2014, (4, – 1), 120]**

(1*) 3bc – 2ad = 0 (2) 3bc + 2ad = 0 (3) 2bc – 3ad = 0 (4) 2bc + 3ad = 0

Sol. 4ax + 2ay + c = 0

$5bx + 2by + d = 0$

$$\frac{x}{2ad-2bc} = \frac{y}{5bc-4ad} = \frac{1}{8ab-10ab}$$

$\Rightarrow$ $x = \frac{bc-ad}{ab}$, $y = \frac{4ad-5bc}{2ab}$

In fourth quadrant point equidistant from axis will have sum of x & y co-ordinate = 0
चतुर्थचतुर्थांश मे अक्षो से बराबर दूरी पर x, y के निर्देशांकों का योगफल होगा = 0

$\Rightarrow$ $\frac{2bc-2ad}{2ab} + \frac{4ad-5bc}{2ab} = 0$

$\Rightarrow$ $2ad - 3bc = 0$

11. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is **[JEE(Main) 2015, (4, – 1), 120]**

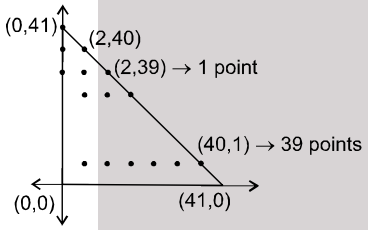
(1) 901 (2) 861 (3) 820 (4) 780

त्रिभुज, जिसके शीर्ष (0, 0), (0, 41) तथा (41, 0) हैं, के आंतरिक भाग में स्थित उन बिन्दुओं की संख्या जिनके दोनों निर्देशांक पूर्णांक है, है– **[JEE(Main) 2015, (4, – 1), 120]**

(1) 901 (2) 861 (3) 820 (4) 780

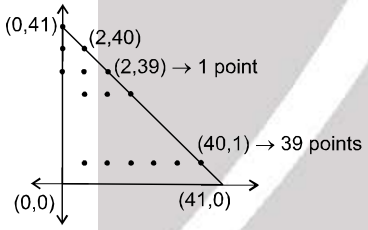
Ans. **(4)**

Sol.



$1 + 2 + \ldots\ldots 39 = \frac{39}{2}(39 + 1) = 780$

Hindi.



$1 + 2 + ... 39 = \frac{39}{2}(39 + 1) = 780$

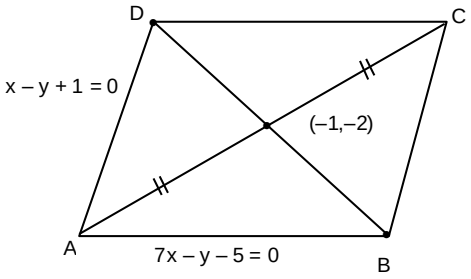
12. Two sides of a rhombus are along the lines, $x – y + 1 = 0$ and $7x – y – 5 = 0$. if its diagonals intersect at $(–1, –2)$, then which one of the following is a vertex of this rhombus ? **[JEE(Main) 2016, (4, – 1), 120]**

यदि एक समचतुर्भज की दो भुजाएँ, रेखाओं $x – y + 1 = 0$ तथा $7x –y –5 = 0$ की दिशा में हैं तथा इसके विकर्ण बिंदु $(–1, –2)$ पर प्रतिच्छेद करते है, तो इस समचतुर्भुज का निम्न में से कौन–सा शीर्ष है ?

(1) $(–3, –8)$ (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) $(–3, –9)$

Ans. **(2)**

Sol.



On solving equation of AB & AD
AB & AD को हल करने पर
vertex A(1, 2)

A(1, 2)

$\because$ P is mid point of AC. Hence vertex C is (–3, –6).

$\because$ P, AC का मध्य बिन्दु है अतः शीर्ष C (–3, –6) है.

So equation of other two sides are $7x – y + 15 = 0$ and $x – y – 3 = 0$.
अतः अन्य दो भुजाओं के समीकरण $7x – y + 15 = 0$ और $x – y – 3 = 0$.

Hence other vertices are $\left(\frac{1}{3},-\frac{8}{3}\right)$ and $\left(-\frac{7}{3},-\frac{4}{3}\right)$

अतः अन्य शीर्ष $\left(\frac{1}{3},-\frac{8}{3}\right)$ और $\left(-\frac{7}{3},-\frac{4}{3}\right)$

13. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main) 2017, (4, – 1), 120]**

माना k एक ऐसा पूर्णांक है कि त्रिभुज, जिसके शीर्ष (k, –3k), (5, k) तथा (–k, 2) है का क्षेत्रफल 28 वर्ग इकाई है, तो त्रिभुज के लंबकेन्द्र जिस बिंदु पर है, वह है: **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(2,-\frac{1}{2}\right)$ (2) $\left(1,\frac{3}{4}\right)$ (3) $\left(1,-\frac{3}{4}\right)$ (4) $\left(2,\frac{1}{2}\right)$

Ans. **(4)**

Sol. $\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$

$k(k – 2) – 5(–3k – 2) – k (–3k – k) = \pm 56$

$k^2 – 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$

$5k^2 + 13k + 10 \pm 56 = 0$

$5k^2 + 13k + 66 = 0$ or या $5k^2 + 13k – 46 = 0$

No solution कोई हल नहीं or या $k = \frac{-13 \pm \sqrt{169 + 920}}{10}$

$k = \frac{-13 \pm 33}{10} \Rightarrow k = 2$ or या $k = -\frac{46}{10}$ (which is not an integer जो कि पूर्णांक नहीं है)

$\therefore$ vertices शीर्ष A(2, – 6), B (5,2), C (–2,2)

Equation of altitude dropped from vertex A is (शीर्ष A से डाले गये लम्ब की समीकरण)

$x = 2$ ..... (i)

Equation of altitude dropped from vertex C is (शीर्ष C से डाले गये लम्ब की समीकरण)

$3x + 8y – 10 = 0$ .....(ii)

solving both (i) and (ii) (i) तथा (ii) को हल करने पर

orthocentre लम्बकेन्द्र $\left(2, \frac{1}{2}\right)$

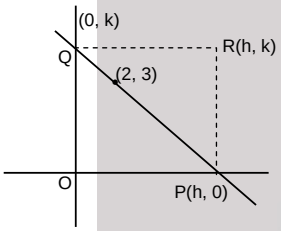
14. A straight line through a fixed point (2,3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

एक सरल रेखा, जो एक अचर बिन्दु (2,3) से होकर जाती है, निर्देशांक अक्षों को दो विभिन्न बिन्दुओं P तथा Q पर प्रतिच्छेद करती है। यदि O मूल बिन्दु है तथा आयत OPRQ को पूरा किया जाता है तो R का बिन्दुपथ है :

Straight Line [JEE(Main) 2018, (4, – 1), 120]

(1*) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

Sol. (1)



$$\begin{vmatrix} 0 & k & 1 \\ 2 & 3 & 1 \\ h & 0 & 1 \end{vmatrix} = 0$$

$– (2 –h) + 1(–3h) = 0$

$–2y + xy –3x = 0$

$3x + 2y = xy$

15. Consider the set all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ? **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) The lines are not concurrent

(2) The lines are all parallel

(3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) Each the line passes through the origin.

ऐसी सभी रेखाओं $px + qy + r = 0$ के समुच्चय पर विचार कीजिए जिनके लिए $3p + 2q + 4r = 0$ है, तो निम्न में से कौन–सा एक कथन सत्य है ?

(1) रेखाएं संगामी नहीं है।

(2) सभी रेखाएँ समांतर हैं।

(3) रेखाएँ बिन्दु $\left(\frac{3}{4}, \frac{1}{2}\right)$ पर संगामी हैं।

(4) प्रत्येक रेखा मूल बिंदु से होकर जाती हैं।

Ans. (3)

Sol. Given relation is $3p + 2q + 4r = 0$

$\Rightarrow \quad \frac{3}{4}p + \frac{q}{2} + r = 0 \quad$ .....(ii)

comparing (1) and (2) we get $x = \frac{3}{4}, y = \frac{1}{2}$

hence we can say that these lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$

Hindi. दिया गया सम्बन्ध $3p + 2q + 4r = 0$

$\Rightarrow \quad \frac{3}{4}p + \frac{q}{2} + r = 0 \quad$ .....(ii)

समीकरण (i) व (ii) की तुलना करने पर $x = \frac{3}{4}, y = \frac{1}{2}$

अतः हम कह सकते है कि दी गई रेखाऐं बिन्दु $\left(\frac{3}{4}, \frac{1}{2}\right)$ पर संगामी है।

16. Let the equations of two sides of a triangle be $3x – 2y + 6 = 0$ and $4x + 5y – 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is :

मना एक त्रिभुज की दो भुजाओं के समीकरण $3x – 2y + 6 = 0$ तथा $4x + 5y – 20 = 0$ है। तथा इस त्रिभुज का लंबकेंद्र (1, 1) पर हैं, तो इसकी तीसरी भुजा का समीकरण है : **[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]**

(1) $26x – 122y – 1675 = 0$ (2) $26x + 61y + 1675 = 0$

(3) $122y – 26x – 1675 = 0$ (4) $122y + 26x + 1675 = 0$

Ans. (1)

Straight line XI T,

Sol. $4x + 5y = 20 \quad$ …….(i)

$3x – 2y + 6 = 0 \quad$ …….(ii)

orthocenter (1, 1)

Line $\perp$ to $4x + 5y = 20$ and passes (1, 1) is $5x – 4y = 1 \quad$ …….(iii)

and line $\perp$ to $3x - 2y + 6 = 0$ and passes (1, 1) is $2x + 3y = 5$ .....(iv)

on solving (i) & (iv) pt $\left(\frac{35}{2}, -10\right)$

on solving (ii) & (iii) pt $\left(-13, -\frac{33}{2}\right)$

side BC is $y + 10 = \frac{-\frac{33}{2}+10}{-13-\frac{35}{2}}\left(x - \frac{35}{2}\right)$

$\Rightarrow$ $y + 10 = \frac{13}{61}\left(x - \frac{35}{2}\right)$ $\Rightarrow 26x - 122y - 1675 = 0$

Hindi. $4x + 5y = 20$ .....(i)

$3x - 2y + 6 = 0$ .....(ii)

लम्बकेन्द्र (1, 1)

रेखा $4x + 5y = 20$ के लम्बवत तथा (1, 1) से गुजरती हो $5x - 4y = 1$ .....(iii)

तथा रेखा $3x - 2y + 6 = 0$ के लम्बवत तथा (1, 1) से गुजरती हो $2x + 3y = 5$ .....(iv)

(i) व (iv) को हल करने पर बिन्दु $\left(\frac{35}{2}, -10\right)$

(ii) व (iii) को हल करने पर बिन्दु $\left(-13, -\frac{33}{2}\right)$

भुजा BC , $y + 10 = \frac{-\frac{33}{2}+10}{-13-\frac{35}{2}}\left(x - \frac{35}{2}\right)$

$\Rightarrow$ $y + 10 = \frac{13}{61}\left(x - \frac{35}{2}\right)$ $\Rightarrow 26x - 122y - 1675 = 0$

17. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

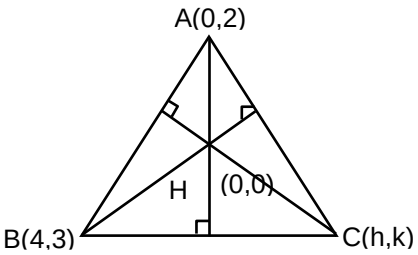
(1) third (2) second (3) first (4) fourth

एक त्रिभुज के दो शीर्ष (0, 2) तथा (4, 3) है। यदि इसका लंबकेद्र मूलबिन्दु पर है, तो इसका तीसरा शीर्ष किस चतुर्थांश में है? **[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]**

(1) तीसरा (2) दूसरा (3) प्रथम (4) चौथा

Ans. (2)

Sol.



Since H is orthocentre of ΔABC

then C is also orthocentre of ΔAHB

equation of line AC is

$4x + 3y = 6$ .....(i)

equation of line BC is

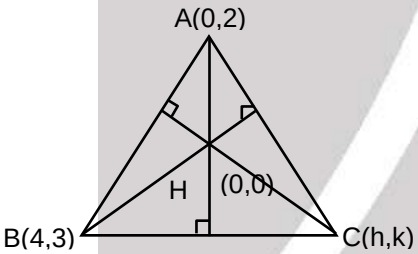
$y = 3$ .....(ii)

on solving (i) and (ii)

$x = -\frac{3}{4}$, $y = 3$

$\Rightarrow$ point lies in 2nd quadrant

Hindi



चूंकि ΔABC का लम्बकेन्द्र H है।

तब ΔAHB का लम्बकेन्द्र C है।

रेखा AC का समीकरण

$4x + 3y = 6$ .....(i)

रेखा BC का समीकरण

$y = 3$ .....(ii)

(i) तथा (ii) हल करने पर

$x = -\frac{3}{4}$, $y = 3$

$\Rightarrow$ बिन्दु 2nd चर्तुथांश में है।

18. Let O(0,0) and A (0,1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

माना O(0,0) तथा A (0,1) दो निश्चित बिन्दु है, तो ऐसे बिंन्दु P जिनके लिए ΔAOP का परिमाप 4, हो, का बिंदुपथ है–

(1) $9x^2 - 8y^2 + 8y = 16$ (2) $8x^2 - 9y^2 + 9y = 18$

(3) $9x^2 + 8y^2 - 8y = 16$ (4) $8x^2 + 9y^2 - 9y = 18$

Ans. (3) **[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120] [Straight Line]**

Sol. $\sqrt{x^2+y^2} + \sqrt{x^2+(y-1)^2} = 3$

$\Rightarrow x^2 + y^2 - 2y + 1 = 9 + x^2 + y^2 - 6\sqrt{x^2+y^2}$

$3\sqrt{x^2+y^2} = 4 + y$

$9x^2 + 9y^2 = 16 + y^2 + 8y$

19. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular form the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is -

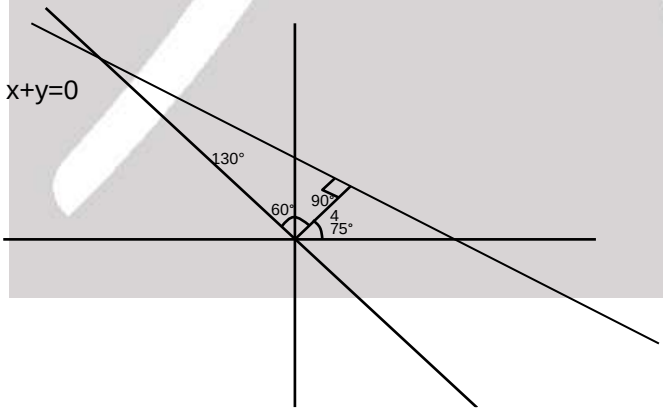
मूलबिन्दु से 4 इकाई की दूरी पर एक सरल रेखा L निर्देशांक अक्षों पर धनात्मक अंतः खण्ड बनाती है तथा मूलबिन्दु से इस रेखा पर लंब, रेखा x + y = 0 के साथ 60º का कोण बनाता है। तो रेखा L का एक समीकरण है :

(1) $\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$ (2) $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

(3) $\sqrt{3}x + y = 8$ (4) $x + \sqrt{3}y = 8$

Ans. (2) **[JEE(Main) 2019, Online (12-04-19),P-2 (4, – 1), 120]**

Sol.



Hence equation of line is $x\cos\theta + y\sin\theta = p$

$x\cos(75°) + y\sin(75°) = 4$

$x\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + y\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = 4$

$$x(\sqrt{3}-1)+y(\sqrt{3}+1)=8\sqrt{2}$$

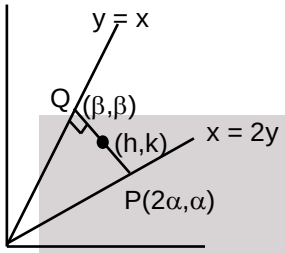
20. The locus of the mid-point of the perpendiculars drawn from points on the line, x = 2y to the line x = y is:

रेखा x = 2y के बिन्दुओं से रेखा x = y पर डाले गये लम्बों के मध्य बिन्दुओं का बिन्दुपथ है:

(1) 3x – 2y = 0 (2) 3x – 3y = 0 (3) 5x – 7y = 0 (4) 7x – 5y = 0

Ans. (3) **[JEE(Main) 2020, Online (07-01-20),P-2 (4, –1), 120]**

Sol.



slope of PQ की प्रवणता = $\frac{k-\alpha}{h-2\alpha}=-1$

$\Rightarrow k-\alpha=-h+2\alpha$

$\Rightarrow \alpha=\frac{h+k}{3}$ .....(1)

Also तथा $2h=2\alpha+\beta$

$2k=\alpha+\beta$

$\Rightarrow 2h=\alpha+2k$

$\Rightarrow \alpha=2h-2k$ .....(2)

from (1) और & (2) से

$\frac{h+k}{3}=2(h-k)$

so locus is अतः बिन्दुपथ $6x-6y=x+y \Rightarrow 5x=7y$

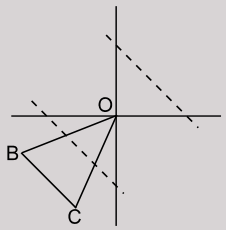
High Level Problems (HLP)

Marked Questions may have for Revision Questions.
चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

1. The vertices of a triangle OBC are O(0,0) B(–3,–1) and C(–1,–3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.
ΔOBC के शीर्ष O(0,0), B(–3,–1) तथा C(–1,–3) है। BC के समान्तर उस सरल रेखा की समीकरण ज्ञात कीजिए जो भुजाओं OB व OC को प्रतिच्छेद करती है तथा जिसकी O(0,0) से लम्बवत् दूरी $\frac{1}{2}$ है–

Ans. $x + y + \frac{1}{\sqrt{2}} = 0.$

Sol. Slope of BC is $= \frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$

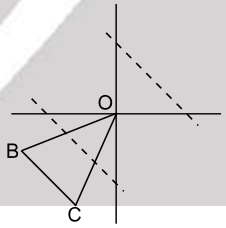


$\therefore$ Equation of a line parallel to BC is
$y = -x + c$ i.e. $x + y - c = 0$
its distnace from the origin is $\left|\frac{c}{\sqrt{2}}\right| = \frac{1}{2}$ $\therefore$ $c = \pm\frac{1}{\sqrt{2}}$

$\therefore$ Equations of the lines are $x + y \pm \frac{1}{\sqrt{2}} = 0$

Since the required line intersects OB and OC, therefore, it is the line whose y intercept is negative.
Hence the required line is $x + y + \frac{1}{\sqrt{2}} = 0.$

Hindi. BC की प्रवणता $= \frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$



$\therefore$ BC के समान्तर रेखा की समीकरण
$y = -x + c$ i.e. $x + y - c = 0$
इसकी मूल बिन्दु (0, 0) से दूरी : $\left|\frac{c}{\sqrt{2}}\right| = \frac{1}{2}$ $\therefore$ $c = \pm\frac{1}{\sqrt{2}}$

$\therefore$ रेखाओं की समीकरणें है $x + y \pm \frac{1}{\sqrt{2}} = 0$

चूंकि अभीष्ट रेखा OB व OC को काटती है अतः यह वह रेखा होगी जिसका y अन्तःखण्ड ऋणात्मक है
अतः अभीष्ट रेखा $x + y + \frac{1}{\sqrt{2}} = 0$ है।

2. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a+b)=ab(x+y)$.

सरल रेखाओं $\frac{x}{a}+\frac{y}{b}=1$ एवं $\frac{x}{b}+\frac{y}{a}=1$, के प्रतिच्छेद बिन्दु से गुजरने वाली एक चर रेखा, निर्देशी अक्षों को क्रमशः A एवं B पर मिलती है। प्रदर्शित कीजिए कि AB के मध्य बिन्दु का बिन्दुपथ $2xy(a+b)=ab(x+y)$ है।

Sol. By family of lines $\left(\frac{x}{a}+\frac{y}{b}-1\right)+\lambda\left(\frac{x}{b}+\frac{y}{a}-1\right)=0$ it cuts x-axis at $A\left(\frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}},\ 0\right)$

y - axis at $B\left(0,\ \frac{\lambda+1}{\frac{\lambda}{a}+\frac{1}{b}}\right)$. Let mid point of AB is P(h, k) $\Rightarrow$ $2h=\frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}}$, $2k=\frac{\lambda+1}{\frac{\lambda}{a}+\frac{1}{b}}$

eliminate λ from both equations we get $2hk(a+b)=ab(h+k)$
Hence locus of P(h, k) is $2xy(a+b)=ab(x+y)$

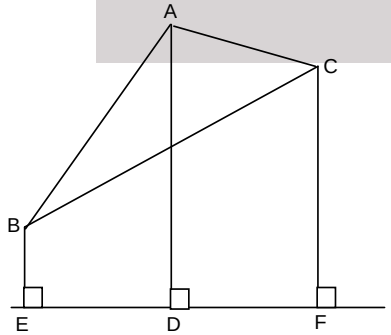
Hindi. रेखाओं निकाय का समीकरण $\left(\frac{x}{a}+\frac{y}{b}-1\right)+\lambda\left(\frac{x}{b}+\frac{y}{a}-1\right)=0$ यह x-अक्ष को $A\left(\frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}},\ 0\right)$ पर काटती है।

y - अक्ष को $B\left(0,\ \frac{\lambda+1}{\frac{\lambda}{a}+\frac{1}{b}}\right)$ पर काटती है। माना कि AB का मध्य बिन्दु P(h, k) है। $2h=\frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}}$, $2k=\frac{\lambda+1}{\frac{\lambda}{a}+\frac{1}{b}}$

दोनों समीकरणों से λ को विलोपित करने पर $2hk(a+b)=ab(h+k)$
अतः P(h, k) का बिन्दुपथ $2xy(a+b)=ab(x+y)$ है।

3. From the vertices A, B, C of a triangle ABC, perpendiculars AD, BE, CF are drawn to any straight line. Show that the perpendiculars from D, E, F to BC, CA, AB respectively are concurrent.
त्रिभुज ABC के शीर्ष A,B,C से लम्ब AD, BE, CF किसी सरल रेखा पर खीचें जाते है। दर्शाइयें कि D, E, F से लम्ब BC, CA और AB क्रमश : संगामी है।

Sol. Without loss of generality, we can take the line as the x-axis and A, B, C as (x_1, y_1), (x_2, y_2) and (x_3, y_3).
The points D, E, F are $(x_1, 0)$, $(x_2, 0)$ and $(x_3, 0)$.
Equation of the perpendicular from D to BC is
$(x-x_1)(x_3-x_2)+y(y_3-y_2)=0$ ……..(L_1)
also, the other two perpendiculars are
$(x-x_2)(x_1-x_3)+y(y_1-y_3)=0$ ……..(L_2)
and $(x-x_3)(x_2-x_1)+y(y_2-y_1)=0$ ……..(L_3)



We have $L_1+L_2+L_3=0$ which implies that the three perpendiculars are concurrent.

Hindi Without loss of generality, we can take the line as the x-axis and A, B, C as (x_1, y_1), (x_2, y_2) and (x_3, y_3).
बिन्दु D, E, F $(x_1, 0)$, $(x_2, 0)$ और $(x_3, 0)$ है।

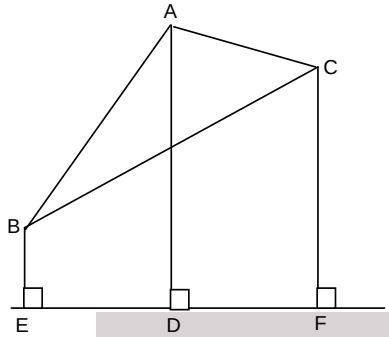
D से भुजा BC पर लम्ब का समीकरण है –

$(x - x_1)(x_3 - x_2) + y(y_3 - y_2) = 0$ ……..(L_1)

तथा अन्य दो लम्बवत् रेखाऐ

$(x - x_2)(x_1 - x_3) + y(y_1 - y_3) = 0$ ……..(L_2)

और $(x - x_3)(x_2 - x_1) + y(y_2 - y_1) = 0$ ……..(L_3)



यहाँ $L_1 + L_2 + L_3 = 0$ का अर्थ है यह है कि तीन लम्बवत् रेखाऐं संगामी है।

4. A triangle is formed by the lines whose equations are AB : x + y – 5 = 0, BC : x + 7y – 7 = 0 and CA : 7x + y + 14 = 0. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

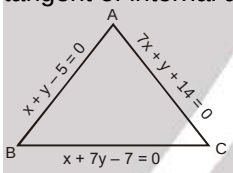
रेखाओं AB : x + y – 5 = 0, BC : x + 7y – 7 = 0 एवं CA : 7x + y + 14 = 0 से त्रिभुज बनाया जाता है। कोण B के अन्तः समद्विभाजक एवं कोण C के बाह्य समद्विभाजक की समीकरण ज्ञात कीजिए। अन्तः कोण A की प्रकृति ज्ञात कीजिए एवं इसके समद्विभाजक की समीकरण ज्ञात कीजिए।

Ans. 3x + 6y – 16 = 0; 8x + 8y + 7 = 0 ; 12x + 6y – 11 = 0

Sol. ∵ The slopes of the lines AB, BC and CA are –1, $-\frac{1}{7}$ and –7 respectively

Let $m_1 = -\frac{1}{7}$, $m_2 = -1$, $m_3 = -7$ ∵ $m_1 > m_2 > m_3$

∴ tangent of internal angles of the triangle are



$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}$, $\tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$, $\tan C = \frac{m_3 - m_1}{1 + m_1 m_3}$

⇒ $\tan A = \frac{3}{4}$, $\tan B = \frac{3}{4}$ and $\tan C = -\frac{24}{7}$

∴ interior angles A and B are acute and interior angle C is obtuse

∴ internal bisector of B = acute bisector of B≡ 3x + 6y – 16 = 0

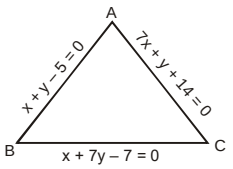
External bisector of C ≡ acute bisector of C ≡ 8x + 8y + 7 = 0

Internal bisector of A ≡ acute bisector of A ≡ 12x + 6y – 11 = 0

Hindi. ∵ सरल रेखाओं AB, BC तथा AC की प्रवणताऐं क्रमशः – 1, $-\frac{1}{7}$ तथा –7 हैं।

माना $m_1 = -\frac{1}{7}$, $m_2 = -1$, $m_3 = -7$ ∵ $m_1 > m_2 > m_3$

∴ त्रिभुज के अन्तःकोणों की स्पर्शज्याओं के मान हैं



$\therefore \quad \tan A = \frac{m_1 - m_2}{1 + m_1 m_2}, \tan B = \frac{m_2 - m_3}{1 + m_2 m_3}, \tan C = \frac{m_3 - m_1}{1 + m_1 m_3}$

$\Rightarrow \quad \tan A = \frac{3}{4}, \tan B = \frac{3}{4}$ and $\tan C = -\frac{24}{7}$

$\therefore$ अन्तःकोण A तथा B न्यूनकोण है तथा अन्तःकोण C अधिक कोण है।

B का अन्तःकोण समद्विभाजक = B का न्यूनकोण समद्विभाजक $\equiv 3x + 6y - 16 = 0$

C का बाह्यकोण समद्विभाजक = C का न्यूनकोण समद्विभाजक $\equiv 8x + 8y + 7 = 0$

A का अन्तःकोण समद्विभाजक = A का न्यूनकोण समद्विभाजक $\equiv 12x + 6y - 11 = 0$

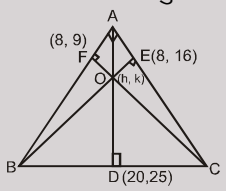
5. The coordinates of the feet of $\perp$ from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). Then find the coordinates of a vertices of the Δ

किसी त्रिभुज के शीर्षों से सम्मुख भुजाओं पर डाले गये लम्बपाद के निर्देशांक (20, 25), (8, 16) एवं (8, 9) है, तो त्रिभुज के किसी शीर्ष के निर्देशांक है–

Ans. (5, 10). (50, – 5), (15, 30)

Sol. Orthocentre O of the ΔABC is the incentre of the pedal ΔDEF.

ΔABC का लम्बकेन्द्र 'O' पदिक त्रिभुज ΔDEF का अन्तःकेन्द्र होगा।



$ED = \sqrt{(20-8)^2 + (25-16)^2} = 15$

$FD = 20, \qquad EF = 7$

$H = \frac{7 \times 20 + 20 \times 8 + 15 \times 8}{7 + 20 + 15} = 10$

$K = \frac{7 \times 25 + 20 \times 16 + 15 \times 9}{7 + 20 + 15} = 15$

0(10, 15)

$AC \equiv y - 2x = 0$

$AB \equiv 3y + x - 35 = 0$

$BC \equiv x + y - 45 = 0 \qquad \Rightarrow \qquad A(5, 10), B(50, -5)C(15, 30)$

6. Let P is any point inside the triangle ABC of side lengths 6, 5, 5 units and p_1, p_2, p_3 be the lengths of perpendiculars drawn from P to the sides of triangle. Find the maximum value of $p_1.p_2.p_3$.

Ans. $p_1 p_2 p_3 \le \frac{256}{75}$

माना लम्बाईयाँ 6, 5, 5 ईकाई की भुजाओं से बने ΔABC के अन्दर है तथा p_1, p_2, p_3 , P से त्रिभुज की भुजाओं पर डाले गए लम्बों की लम्बाईयां है। $p_1.p_2.p_3$ का अधिकतम मान है –

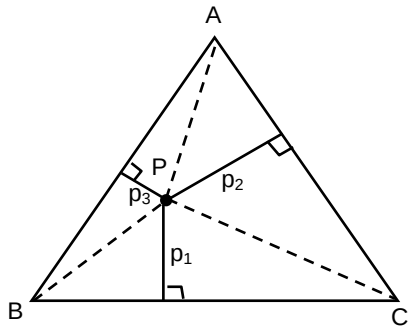
Ans. $p_1 p_2 p_3 \le \frac{256}{75}$

Sol. Area क्षेत्रफल ΔABC = Area क्षेत्रफल ΔBCP + Area क्षेत्रफल ΔCPA + Area क्षेत्रफल ΔAPB

$12 = 3p_1 + \frac{5}{2}p_2 + \frac{5}{2}p_3$

Applying AM $\ge$ G.M से

$$\frac{3p_1+\frac{5}{2}p_2+\frac{5}{3}p_3}{3} \geq \left((3p_1)\left(\frac{5}{2}p_2\right)\left(\frac{5}{2}p_3\right)\right)^{1/3}$$



$$\frac{12}{3} \geq \left(\frac{75}{4}p_1p_2p_3\right)^{1/3}$$

$$p_1p_2p_3 \leq \frac{256}{75}.$$

7. Let in ΔPAB, A is (0, 0), B is (a, 0) and P is variable such that $\angle PBA$ is equal to three times $\angle PAB$ the, find the locus of P.

Ans. $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0$.

मानाकि ΔPAB में A(0, 0), B(a, 0) है। तथा P एक चर रेखा है जो कि इस प्रकार है कि $\angle PBA$ का मान $\angle PAB$ के मान का तीन गुने के बराबर है तब बिन्दु P का बिन्दुपथ है।

Ans. $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0$.

Sol. Let माना $A \equiv (0, 0), B \equiv (a, 0)$

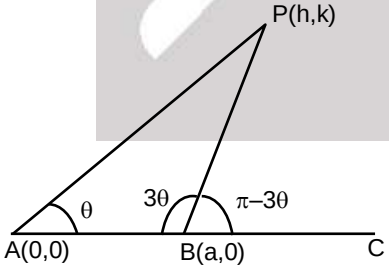
P (h, k), $\angle PAB = \theta$

$\angle PBA = 3\theta$

Now अब $\tan\theta = \frac{k}{h}$ … (1)

$$\tan(\pi - 3\theta) = \frac{k}{h-a}$$

$$\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \frac{-k}{h-a}$$



From (1) से,

$$\frac{\frac{3k}{h}-\frac{k^3}{h^3}}{1-\frac{3k^2}{h^2}} = \frac{-k}{h-a} \Rightarrow \frac{3h^2-k^2}{h(h^2-3k^2)} = \frac{-1}{h-a}$$

$\Rightarrow 3h^3 - k^2h - 3ah^2 + ak^2 = -h^3 + 3k^2h$

$4h^3 - 4k^2h - 3ah^2 + ak^2 = 0$

Locus of P(h, k) बिन्दुपथ is $4x^3 - 4xy^2 - 3ax^2 + ay^2 = 0$.

8. Through a fixed point any straight line is drawn meeting two given parallel straight lines in P and Q, through P and Q straight lines are drawn in fixed directions, meeting in R. Prove that the locus of R is straight line.

स्थिर बिन्दु से कोई सरल रेखा खीचीं जाती है जो दी गई दो समान्तर सरल रेखाओं को P तथा Q पर मिलती है, P और Q से सरल रेखाऐ एक निश्चित दिशा में खीचीं जाती है, जो R पर मिलती है, दर्शाइये कि R का बिन्दुपथ एक सरल रेखा है।

Sol. Take the fixed point O for origin, and the axis of y parallel to the two parallel straight lines, and let the equations of these parallel lines be x = a, x = b.

Then, if the equation of OPQ be y = mx, the abscissa of Q is b, and therefore its ordinate mb. Let PR be always parallel to y = m'x and QR always parallel to y = m"x, then the equation of PR, QR will be

$y - ma = m'(x - a)$ .....(i) $y - mb = m''(x - b)$ .....(ii)

To find locus we have only to eliminate m between the equation (i) and (ii)

The result is $(b - a)\, y = m'b(x - a) - m''\, a\, (x - b)$.

This equation is of the first degree, and therefore the required locus is a straight line.

Hindi. मूलबिन्दु के लिए एक स्थिर बिन्दु O लीजिए तथा y अक्ष के समान्तर दो समान्तर सरल रेखाएं है माना इन समान्तर सरल रेखाओं का समीकरण x = a और x = b है।

तब यदि OPQ का समीकरण y = mx है। Q का भुज b है इसलिए इसकी कोटि mb है। माना PR सदैव y = m'x के समान्तर है तथा QR, y = m"x के समान्तर है तब PR, QR का समीकरण होगा

$y - ma = m'(x - a)$ .....(i) $y - mb = m''(x - b)$ .....(ii)

बिन्दुपथ ज्ञात करने के लिए समीकरण (i) और (ii) से कवल m को विलोपित करते है।

अभीष्ट परिणाम $(b - a)\, y = m'b(x - a) - m''\, a\, (x - b)$.

इसलिए समीकाण प्रथम घात की है और अभीष्ट बिन्दुपथ एक सरल रेखा है।

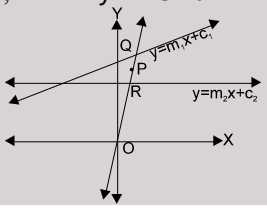
9. Through the origin O a straight line is drawn to cut the lines $y = m_1x + C_1$ and $y = m_2x + C_2$ at Q and R. respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean of OQ and OR.

मूल बिन्दु O से जाने वाली एक रेखा इस प्रकार है कि यह रेखाओं $y = m_1x + C_1$ तथा $y = m_2x + C_2$ को क्रमशः Q व R पर काटती है। इस चर रेखा पर स्थित किसी बिन्दु P का बिन्दुपथ ज्ञात कीजिए यदि OP, OQ व OR का गुणोत्तर माध्य है।

Ans. $\mathbf{(y - m_1x)\,(y - m_2x) = c_1c_2}$

Sol. Let the line (L) through the origin is

$x = r\cos\theta$; $y = r\sin\theta$



as L intersects L_1 at Q and $OQ = r_1$

$\therefore$ $r_1 \sin\theta = m_1 r_1 \cos\theta + c_1$ .....(1)

similarly, L intersects L_2 at R and $OR = r_2$

$r_2 \sin\theta = m_2 r_2 \cos\theta + c_2$ .....(2)

Let $P \equiv (h, k)$ & $OP = r$

$\therefore$ $r^2 = r_1 r_2$ .....(3)

& $h = r\cos\theta$ .....(4)

$k = r\sin\theta$ .....(5)

putting the values of r_1 and r_2 from (1) and (2) in (3)

$\therefore$ $r^2 = \dfrac{c_1}{(\sin\theta - m_1\cos\theta)} \dfrac{c_2}{(\sin\theta - m_2\cos\theta)}$. .....(6)

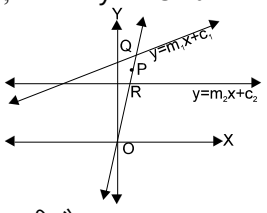
putting the value of $\cos\theta$ and $\sin\theta$ from (4) and (5) in (6), we get

$\Rightarrow \quad r^2 = \dfrac{c_1 c_2}{\left(\dfrac{k}{r} - m_1\dfrac{h}{r}\right)\left(\dfrac{k}{r} - m_2\dfrac{h}{r}\right)} \quad \Rightarrow \quad (k - m_1h)(k - m_2h) = c_1c_2$

replacing (h, k) by (x, y) we get the desired locus as $(y - m_1x)(y - m_2x) = c_1c_2$

Hindi माना मूल बिन्दु से जाने वाली रेखा (L) है।

$x = r\cos\theta \quad ; \quad y = r\sin\theta$



चूंकि $L = 0$, L_1 को Q पर काटती है तथा $OQ = r_1$

$\therefore \quad r_1\sin\theta = m_1r_1\cos\theta + c_1$ .....(1)

इसी प्रकार $L = 0$, L_2 को R पर काटती है तथा $OR = r_2$

$r_2\sin\theta = m_2r_2\cos\theta + c_2$ .....(2)

माना $P \equiv (h, k)$ & $OP = r$

$\therefore \quad r^2 = r_1r_2$ .....(3)

तथा $h = r\cos\theta$ .....(4)

$k = r\sin\theta$ .....(5)

(1) व (2) से r_1 व r_2 के मान (3) में रखने पर

$\therefore \quad r^2 = \dfrac{c_1}{(\sin\theta - m_1\cos\theta)} \cdot \dfrac{c_2}{(\sin\theta - m_2\cos\theta)}$ . .....(6)

(4) व (5) से $\cos\theta$ व $\sin\theta$ के मान (6) में रखने पर

$\Rightarrow \quad r^2 = \dfrac{c_1 c_2}{\left(\dfrac{k}{r} - m_1\dfrac{h}{r}\right)\left(\dfrac{k}{r} - m_2\dfrac{h}{r}\right)} \quad \Rightarrow \quad (k - m_1h)(k - m_2h) = c_1c_2$

(h, k) को (x, y) से प्रतिस्थापित करने पर हमें अभीष्ट बिन्दुपथ प्राप्त होता है जो है $(y - m_1x)(y - m_2x) = c_1c_2$.

10. The sides of a triangle are $L_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0$ for r = 1, 2, 3.
Show that its orthocentre is given by
$L_1\cos(\alpha_2 - \alpha_3) = L_2\cos(\alpha_3 - \alpha_1) = L_3\cos(\alpha_1 - \alpha_2)$.
त्रिभुज की भुजाएं $L_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0$, r = 1, 2, 3 के लिए
दर्शाइये कि इसका लम्बकेन्द्र $L_1\cos(\alpha_2 - \alpha_3) = L_2\cos(\alpha_3 - \alpha_1) = L_3\cos(\alpha_1 - \alpha_2)$ होगा।

Sol. Let BC be $L_1 \equiv x\cos\alpha_1 + y\sin\alpha_1 - p_1 = 0$, CA be $L_2 \equiv 0$ and AB be $L_3 = 0$
Then the altitude AD through A is of the form

$L_2 + \lambda L_3 = (x\cos\alpha_2 + y\sin\alpha_2 - p_2) + \lambda(x\cos\alpha_3 + y\sin\alpha_3 - p_3) = 0$

i.e., AD is given by $(\cos\alpha_2 + \lambda\cos\alpha_3)x + (\sin\alpha_2 + \lambda\sin\alpha_3)y - (p_2 + \lambda p_3) = 0$.

Slope of AD $= -\dfrac{\cos\alpha_2 + \lambda\cos\alpha_3}{\sin\alpha_2 + \lambda\sin\alpha_3}$. Now AD $\perp$ BC given

$$-\left(\frac{\cos\alpha_2 + \lambda\cos\alpha_3}{\sin\alpha_2 + \lambda\sin\alpha_3}\right)(-\cot\alpha_1) = -1,$$

or $\cos\alpha_1(\cos\alpha_2 + \lambda\cos\alpha_3) + \sin\alpha_1(\sin\alpha_2 + \lambda\sin\alpha_3) = 0$,

This gives $(\cos\alpha_1\cos\alpha_2 + \sin\alpha_1\sin\alpha_2) + \lambda(\cos\alpha_1\cos\alpha_3 + \sin\alpha_1\sin\alpha_3) = 0$

or equivalents $\lambda = -\dfrac{\cos(\alpha_1 - \alpha_2)}{\cos(\alpha_3 - \alpha_1)}$

$\therefore$ Equation to AD is $L_2 - \frac{\cos(\alpha_1 - \alpha_2)}{\cos(\alpha_3 - \alpha_1)} L_3 = 0$

or equivalently $L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

Similarly the altitude BE is $L_3 \cos(\alpha_1 - \alpha_2) = L_1 \cos(\alpha_2 - \alpha_3)$.

$\therefore$ The orthocentre is given by

$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

Hindi. माना BC, $L_1 \equiv x \cos\alpha_1 + y \sin\alpha_1 - p_1 = 0$, CA , $L_2 \equiv 0$ और AB , $L_3 = 0$ है।

तब A से जाने वाले शीर्षलम्ब AD

$L_2 + \lambda L_3 = (x\cos\alpha_2 + y \sin\alpha_2 - p_2) + \lambda (x \cos\alpha_3 + y \sin\alpha_3 - p_3) = 0$

अर्थात, AD दिया गया है $(\cos\alpha_2 + \lambda \cos\alpha_3) x + (\sin\alpha_2 + \lambda \sin\alpha_3) y - (p_2 + \lambda p_3) = 0$.

AD की प्रवणता $= - \frac{\cos\alpha_2 + \lambda\cos\alpha_3}{\sin\alpha_2 + \lambda\sin\alpha_3}$. अब AD $\perp$ BC है।

$$- \left(\frac{\cos\alpha_2 + \lambda\cos\alpha_3}{\sin\alpha_2 + \lambda\sin\alpha_3} \right) (-\cot\alpha_1) = -1,$$

या $\cos\alpha_1 (\cos\alpha_2 + \lambda \cos\alpha_3) + \sin\alpha_1 (\sin\alpha_2 + \lambda \sin\alpha_3) = 0$,

यह देता है $(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2) + \lambda (\cos\alpha_1 \cos\alpha_3 + \sin\alpha_1 \sin\alpha_3) = 0$

या तुल्य $\lambda = - \frac{\cos(\alpha_1 - \alpha_2)}{\cos(\alpha_3 - \alpha_1)}$

$\therefore$ AD का समीकरण $L_2 - \frac{\cos(\alpha_1 - \alpha_2)}{\cos(\alpha_3 - \alpha_1)} L_3 = 0$ है।

या तुल्य $L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

इसप्रकार शीर्षलम्ब BE $L_3 \cos(\alpha_1 - \alpha_2) = L_1 \cos(\alpha_2 - \alpha_3)$ है।

$\therefore$ लम्बकेन्द्र दिया गया है

$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

या तुल्य $L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

इसप्रकार शीर्षलम्ब BE $L_3 \cos(\alpha_1 - \alpha_2) = L_1 \cos(\alpha_2 - \alpha_3)$ है।

$\therefore$ लम्बकेन्द्र दिया गया है

$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2)$.

11. A line passes through a fixed point R intersecting a fixed line at P. A point Q on RP such that $\frac{RP}{RQ}$ is constant. Then show that locus of Q is a straight line.

एक रेखा स्थिर बिन्दु R से स्थिर रेखा को P पर प्रतिच्छेद करती है। एक बिन्दु Q, RP रेखा पर इस प्रकार है कि $\frac{RP}{RQ}$ अचर है तब दर्शाइयें कि Q का बिन्दुपथ एक सरल रेखा है।

Sol. Let a fixed point $R(\alpha, \beta)$ and fixed line is $ax + by + c = 0$

Equation of line RP is $\frac{x - \alpha}{\cos\theta} = \frac{y - \beta}{\sin\theta} = r$

Coordinate of $P(\alpha + r\cos\theta, \beta + r\sin\theta)$. It lies on line

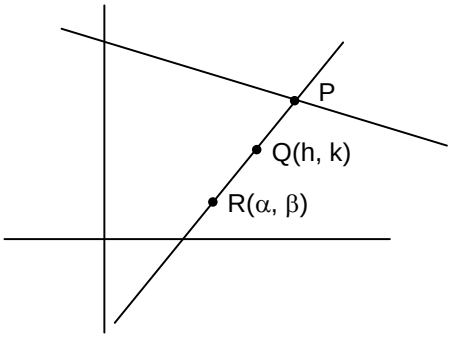
$ax + by + c = 0$

$a(\alpha + r\cos\theta) + b(\beta + r\sin\theta) + c = 0$ …(i)

Let coordinate of Q(h, k) be

$h = \alpha + r_1 \cos\theta$ and $k = \beta + r_1 \sin\theta$

Given that $\frac{r}{r_1} = \lambda$ (constant)



$h-\alpha=\frac{r}{\lambda}\cos\theta$ and $k-\beta=\frac{r}{\lambda}\sin\theta \Rightarrow r\cos\theta=\lambda(h-\alpha)$ and $r\sin\theta=\lambda(k-\beta)$

From equation (i), $a(\alpha+\lambda(h-\alpha))+b(\beta+\lambda(k-\beta))+c=0$

Hence locus of Q is

$a\lambda(x-\alpha)+b\lambda(y-\beta)+a\alpha+b\beta+c=0$.

Hindi माना R(α, β) स्थिर बिन्दु है तथा स्थिर रेखा ax + by + c = 0 है।

रेखा RP का समीकरण $\frac{x-\alpha}{\cos\theta}=\frac{y-\beta}{\sin\theta}=r$ है।

P के निर्देशांक (α + rcosθ, β + rsinθ) है यह रेखा

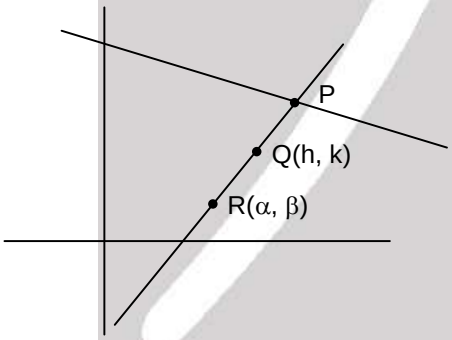
ax + by + c = 0

$a(\alpha+r\cos\theta)+b(\beta+r\sin\theta)+c=0$ ...(i)

माना Q(h, k) के निर्देशांक

$h = \alpha + r_1\cos\theta$ और $k = \beta + r_1\sin\theta$ है।

दिया गया है $\frac{r}{r_1}=\lambda$ (अचर)



$h-\alpha=\frac{r}{\lambda}\cos\theta$ and $k-\beta=\frac{r}{\lambda}\sin\theta \Rightarrow r\cos\theta=\lambda(h-\alpha)$ and $r\sin\theta=\lambda(k-\beta)$

समीकरण (i) से $a(\alpha+\lambda(h-\alpha))+b(\beta+\lambda(k-\beta))+c=0$

अतः Q का बिन्दुपथ है

$a\lambda(x-\alpha)+b\lambda(y-\beta)+a\alpha+b\beta+c=0$.

12. A triangle ABC with a = 8, b = 6 and c = 10 slides on the coordinate axes with vertices A and B on the x–axis and the y–axis respectively. Find the locus of the vertex C.
एक त्रिभुज ABC जिसमें a = 8, b = 6 और c = 10 निर्देशांक अक्षों पर शीर्ष A और B पर क्रमशः x-अक्ष और y-अक्ष पर फिसलती है तब शीर्ष C का बिन्दुपथ ज्ञात कीजिए।

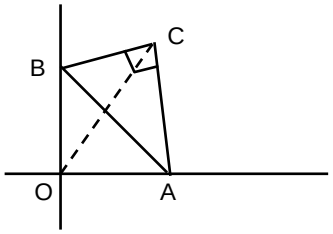
Sol. Since $c^2 = a^2 + b^2$,
ΔABC is right angled at C

$\Rightarrow$ OACB is a cyclic quadrilateral.

$\Rightarrow \angle COB = \angle BAC$

In ΔABC,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 100 - 64}{2 \times 6 \times 10} = \frac{72}{120} = \frac{3}{5}$$



$$\Rightarrow \cos(\Delta COB) = \frac{3}{5} \Rightarrow \cos\left(\frac{\pi}{2} - \angle COA\right) = \frac{3}{5} \Rightarrow \sin(\angle COA) = \frac{3}{5} \Rightarrow \tan(\angle COA) = \frac{3}{4}$$

$\Rightarrow \angle COA$ is fixed $\Rightarrow$ vertex C lies on the line $\frac{y}{x} = \frac{3}{4}$ or $3x - 4y = 0$.

Hindi चूंकि $c^2 = a^2 + b^2$,

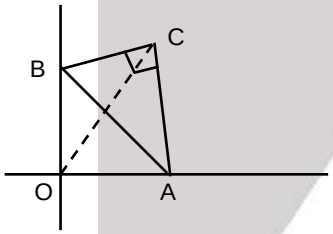
ΔABC, C पर समकोण है।

$\Rightarrow$ OACB चक्रीय चतुर्भुज है।

$\Rightarrow \angle COB = \angle BAC$

ΔABC में

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 100 - 64}{2 \times 6 \times 10} = \frac{72}{120} = \frac{3}{5}$$



$$\Rightarrow \cos(\Delta COB) = \frac{3}{5} \Rightarrow \cos\left(\frac{\pi}{2} - \angle COA\right) = \frac{3}{5} \Rightarrow \sin(\angle COA) = \frac{3}{5} \Rightarrow \tan(\angle COA) = \frac{3}{4}$$

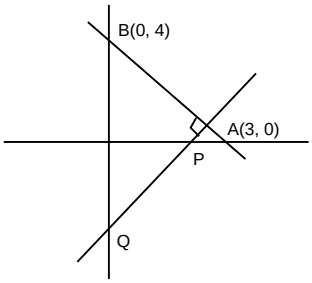
$\Rightarrow \angle COA$ स्थिर है $\Rightarrow$ शीर्ष C, रेखा $\frac{y}{x} = \frac{3}{4}$ या $3x - 4y = 0$ पर स्थित है।

13. The line $L_1 \equiv 4x + 3y - 12 = 0$ intersects the x and the y-axis at A and B respectively. A variable line perpendicular to L_1 intersects the x and the y-axis at P and Q respectively. Find the locus of the circumcentre of triangle ABQ.

रेखा $L_1 \equiv 4x + 3y - 12 = 0$, x अंक्ष और y-अंक्ष को A और B पर क्रमशः प्रतिच्छेद करता है। एक चर रेखा L_1 के लम्बवत् x और y-अंक्ष को क्रमशः P और Q पर मिलती है। त्रिभुज ABQ के परिकेन्द्र का बिन्दुपथ ज्ञात कीजिए।

Sol. Clearly circumcentre of triangle ABQ will lie on the perpendicular bisector of line AB.

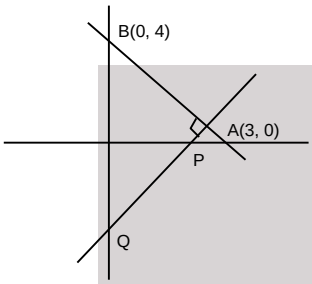
Now equation of perpendicular bisector of line AB is $3x - 4y + \frac{7}{2} = 0$.



Hence locus of circumcentre is 6x – 8y + 7 = 0.

Hindi स्पष्टतयाः त्रिभुज ABQ का परिकेन्द्र, रेखा AB के लम्ब अर्द्धक पर होगा

अब रेखा AB के लम्बवत् अर्द्धक का समीकरण $3x - 4y + \frac{7}{2} = 0$ है।



अतः परिकेन्द्र का बिन्दुपथ 6x – 8y + 7 = 0 है।

14. Show that the orthocentre of Δ formed by the straight lines, $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is a point (x', y') such that $\frac{x'}{\ell} = \frac{y'}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$.

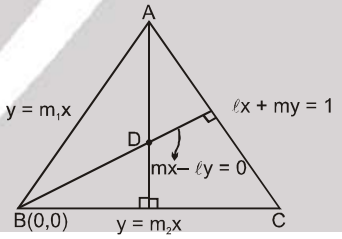
दर्शाइये सरल रेखाओं $ax^2 + 2hxy + by^2 = 0$ और $\ell x + my = 1$ से बने Δ का लम्बकेन्द्र एक बिन्दु (x', y') है जहाँ $= \frac{x'}{\ell} = \frac{y'}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$.

Sol. $ax^2 + 2hxy + by^2 \equiv b(y - m_1 n)(y - m_2 n)$

$m_1 + m_2 = \frac{-2h}{b}$, $m_1 m_2 = \frac{a}{b}$

The line $\ell x + my = 1$ cuts $y = m_1 x$.

रेखा $\ell x + my = 1$, $y = m_1 x$ को प्रतिच्छेदी करती है।



$A\left(\frac{1}{\ell + mm_1}, \frac{m_1}{\ell + mm_1}\right)$

Equation of line AD $\Rightarrow y - \frac{m_1}{\ell + mm_1} = -\frac{1}{m_2}\left(x - \frac{1}{\ell + mm_1}\right)$

रेखा AD का समीकरण $y - \frac{m_1}{\ell + mm_1} = -\frac{1}{m_2}\left(x - \frac{1}{\ell + mm_1}\right)$

$x(\ell + mm_1) - 1 + m_2\{y(\ell + mm_1) - m_1\} = 0$ .....(1)

orthocentre also lies on $mx - \ell y = 0$ or $\frac{x}{\ell} = \frac{y}{m} = \lambda$

लम्बकेन्द्र रेखा $mx - \ell y = 0$ या $\frac{x}{\ell} = \frac{y}{m} = \lambda$ पर स्थित है

put $x = \lambda\ell$ and $y = m\lambda$ in (1)

$x = \lambda\ell$ तथा $y = m\lambda$, (1) रखने पर

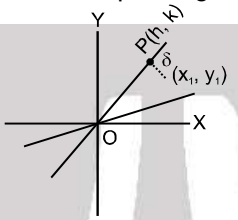
$\therefore \quad \lambda = \frac{a+b}{b\ell^2 - 2h\ell m + am^2}$

15. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co–ordinates is δ, find the combined equation of these straight lines.

मूल बिन्दु से गुजरने वाली दो सरल रेखाओं से बिन्दु (x_1, y_1) की दूरी δ है। इन सरल रेखाओं की संयुक्त समीकरण ज्ञात कीजिए।

Ans. $(y_1^2 - \delta^2)\, x^2 - 2\, x_1 y_1\, xy + (x_1^2 - \delta^2)\, y^2 = 0$

Sol. Let P(h, k) be a variable point on the lines passing through the origin.

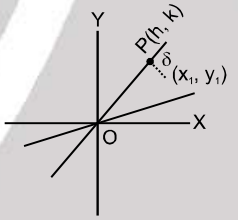


$\therefore \quad \left|\frac{kx_1 - hy_1}{\sqrt{h^2+k^2}}\right| = \delta \quad \Rightarrow \quad (kx_1 - hy_1)^2 = \delta^2 (h^2 + k^2)$

$\therefore$ locus of P(h, k) is $(x_1 y - x y_1)^2 = \delta^2 (x^2 + y^2)$

solving it, we get $(y_1^2 - \delta^2)\, x^2 - 2x_1 y_1\, xy + (x_1^2 - \delta^2)\, y^2 = 0$.

Hindi. माना P(h, k) चर बिन्दु है जोकि मूलबिन्दु से गुजरने वाली रेखाओं पर है।



$\therefore \quad \left|\frac{kx_1 - hy_1}{\sqrt{h^2+k^2}}\right| = \delta \quad \Rightarrow \quad (kx_1 - hy_1)^2 = \delta^2 (h^2 + k^2)$

$\therefore$ P(h, k) का बिन्दुपथ $(x_1 y - x y_1)^2 = \delta^2 (x^2 + y^2)$ है।

हल करने पर हम पातें हैं। $(y_1^2 - \delta^2)\, x^2 - 2x_1 y_1\, xy + (x_1^2 - \delta^2)\, y^2 = 0$.

16. A point moves so that the distance between the feet of the perpendiculars from it on the lines $ax^2 + 2h\, xy + by^2 = 0$ is a constant 2 d. Show that the equation to its locus is,

$(x^2 + y^2)(h^2 - ab) = d^2 \{(a - b)^2 + 4h^2\}$.

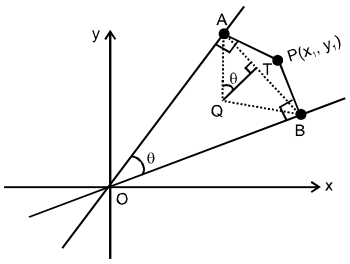
एक बिन्दु इस प्रकार गति करता है कि इससे रेखाओं $ax^2 + 2h\, xy + by^2 = 0$ पर डाले गए लम्ब के पादों के मध्य दूरी 2 d (अचर) है। प्रदर्शित कीजिए कि इसके बिन्दुपथ की समीकरण $(x^2 + y^2)(h^2 - ab) = d^2 \{(a - b)^2 + 4h^2\}$ है।

Sol. $\because$ AB = 2d

$\because$ OAPB is a cyclic quadrilateral and OP will be diameter of the circumcircle of this quadrilateral Let Q be the centre of the circle

$\therefore$ in ΔAQT

$\sin\theta = \frac{2d}{\sqrt{x_1^2 + y_1^2}}$ ..... (i)



$\because$ $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$ .... (ii)

$\therefore$ from (i) and (ii), we get $\dfrac{2\sqrt{h^2 - ab}}{a+b} = \dfrac{2d}{\sqrt{x_1^2 + y_1^2 - 4d^2}}$

$\therefore$ Locus of $P(x_1, y_1)$ is $(x^2 + y^2)(h^2 - ab) = d^2\{(a-b)^2 + 4h^2\}$

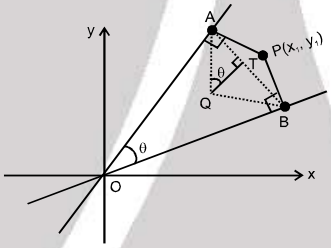
Hindi. $\because$ AB = 2d

$\because$ OAPB एक चक्रीय चतुर्भुज है और OP इस चतुर्भुज के परिवृत्त का व्यास है।
माना Q वृत्त का केन्द्र है।

$\therefore$ ΔAQT में

$\therefore$ in ΔAQT

$\sin\theta = \dfrac{2d}{\sqrt{x_1^2 + y_1^2}}$ ..... (i)



$\because$ $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$ .... (ii)

$\therefore$ (i) व (ii) से, हम पाते है $\dfrac{2\sqrt{h^2 - ab}}{a+b} = \dfrac{2d}{\sqrt{x_1^2 + y_1^2 - 4d^2}}$

$\therefore$ $P(x_1, y_1)$ का बिन्दुपथ है। $P(x_1, y_1)$ is $(x^2 + y^2)(h^2 - ab) = d^2\{(a-b)^2 + 4h^2\}$

17. Show that the pair of lines given by $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$ is equally inclined to the pair given by $ax^2 + 2hxy + by^2 = 0$.
सिद्ध कीजिऐ कि $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$ द्वारा प्रदर्शित सरल रेखाँऐं, $ax^2 + 2hxy + by^2 = 0$ द्वारा प्रदर्शित सरल रेखाओं पर समान कोण पर झुकी हुई है।

Sol. Given pair of lines are $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0 \Rightarrow ax^2 + 2hxy + by^2 = 0$

Equation of pair of bisectors of first pair is $\dfrac{x^2 - y^2}{a^2 - b^2} = \dfrac{xy}{h(a+b)} \Rightarrow \dfrac{x^2 - y^2}{a-b} = \dfrac{xy}{h}$

Which is also bisector of second pair. Hence both pair are equally inclined.

Hindi. दिया गया रेखा युग्म $a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$ व $ax^2 + 2hxy + by^2 = 0$ है।

पहले युग्म का युग्मअर्द्धक का समीकरण $\dfrac{x^2 - y^2}{a^2 - b^2} = \dfrac{xy}{h(a+b)} \Rightarrow \dfrac{x^2 - y^2}{a-b} = \dfrac{xy}{h}$

जोकि दूसरे युग्म का भी अर्द्धक है अतः दोनों युग्म बराबर झुके हुए हैं।

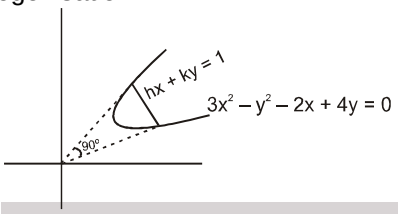
18. All the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrence.

मूल बिन्दु पर समकोण बनाने वाली, वक्र $3x^2 - y^2 - 2x + 4y = 0$ की सभी जीवाँऐ संगामी है। क्या यह परिणाम वक्र $3x^2 + 3y^2 - 2x + 4y = 0$ के लिये भी सत्य है ? यदि हाँ, तो वह बिन्दु ज्ञात कीजिऐ जिस पर ये जीवाँऐ संगामी है।

Ans. **(1, – 2), yes हाँ (1/3, – 2/3)**

Sol. Let equation of chords $hx + ky = 1$

By homogenisation



$3x^2 - y^2 - 2x(hx + ky) + 4y(hx + ky) = 0$

$\therefore$ it makes 90º. Hence

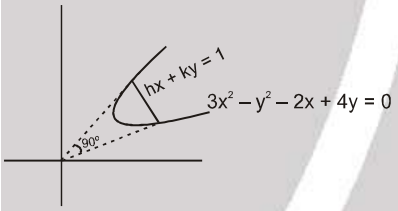
coeff. x^2 + coeff. $y^2 = 0$

$3 - 2h - 1 + 4k = 0 \Rightarrow h - 2k = 1$

Hence all chords are concurrent at (1, – 2)

Hindi. माना जीवाओं की समीकरणे $hx + ky = 1$ है।

समघातीकरण करने पर



$3x^2 - y^2 - 2x(hx + ky) + 4y(hx + ky) = 0$

$\therefore$ यह 90º बनाता है. अतः x^2 का गुणांक + y^2 का गुणांक = 0

$3 - 2h - 1 + 4k = 0 \Rightarrow h - 2k = 1$

अतः सभी जीवाएँ (1, – 2) पर संगामी है।

19. The straight lines $(A^2 - 3B^2)x^2 + 8AB\,xy + (B^2 - 3A^2)y^2 = 0$ form a Δ with the line $Ax + By + C = 0$, then prove that

सरल रेखाएं $(A^2 - 3B^2)x^2 + 8AB\,xy + (B^2 - 3A^2)y^2 = 0$ रेखा $Ax + By + C = 0$ के साथ एक त्रिभुज का निर्माण करती है तब सिद्ध कीजिए

(i) Area of Δ is $\dfrac{C^2}{\sqrt{3}\left(A^2 + B^2\right)}$ त्रिभुज का क्षेत्रफल $\dfrac{C^2}{\sqrt{3}\left(A^2 + B^2\right)}$

(ii) Δ is equilateral त्रिभुज समबाहु है।

(iii) The orthocentre of Δ does not lie on one of its vertexs

त्रिभुज का लम्बकेन्द्र, इसके शीर्षो में से एक नहीं है।

Sol. Let $y = mx$ .....(i)

be a line through the origin making an angle of 60^0 with the line

$Ax + By + C = 0$ .....(ii)

Then we have $\tan 60^0 = \pm\dfrac{m - (-A/B)}{1 + m(-A/B)}$

From (i) we have $m = y/x$. Substituting this value of m in the above result

i.e. $3(B - Am)^2 = (mB + A)^2$, we have

$(A^2 - 3B^2)x^2 + 8ABxy + (B^2 - 3A^2)y^2 = 0$, . . . (iii)

These are the straight lines

through the origin making angles of 60^0 with (ii) i.e. forming an equilateral triangle with (ii).

Now OL = length of perpendicular to $Ax + By + C = 0$ from (0, 0)

$= \frac{C}{\sqrt{(A^2+B^2)}}$ So Area $= \frac{C^2}{\sqrt{3(A^2+B^2)}}$

Hindi. माना $y = mx$ .....(i)

मूलबिन्दु से जाने वाली रेखा है जो रेखा

$Ax + By + C = 0$ के साथ 60^0 का कोण बनाती है। .....(ii)

तब $\tan 60^0 = \pm\frac{m-(-A/B)}{1+m(-A/B)}$

(i) से $m = y/x$, m का मान उपरोक्त परिणाम में रखने पर

अर्थात $3(B - Am)^2 = (mB + A)^2$, we have

$(A^2 - 3B^2)x^2 + 8ABxy + (B^2 - 3A^2)y^2 = 0$, . . . (iii)

ये सरल रेखाएं है।

मूलबिन्दु से जाने वाली (ii) के साथ 60° का कोण बनाती है अर्थात (ii) के साथ समबाहु त्रिभुज बनाती है।

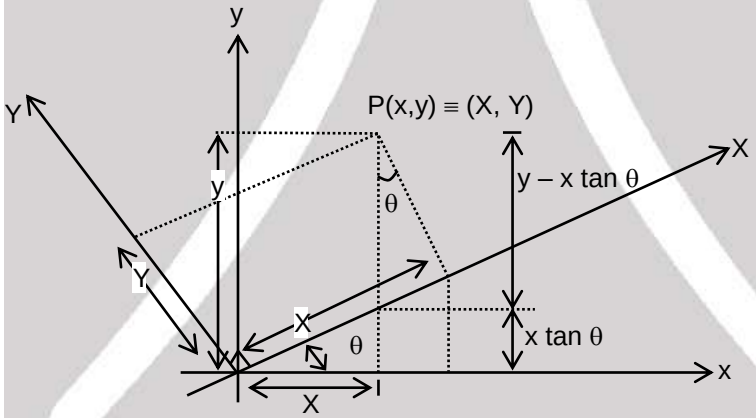
अब OL = (0, 0) से $Ax + By + C = 0$ पर लम्ब की लम्बाई

$= \frac{C}{\sqrt{(A^2+B^2)}}$ इसलिए क्षेत्रफल $= \frac{C^2}{\sqrt{3(A^2+B^2)}}$

Comprehension (Q. 20 & 21)

If coordinate system xy is being rotated through an angle θ in anti clock wise direction about the origin as shown in the diagram, Coordinates of P(x, y) has been change to P(X, Y) in new coordinate system XY, then x, y, X, Y are related as given below.

यदि निर्देशांक पद्धति xy को वामावर्त दिशा में मूल बिन्दु के सापेक्ष θ कोण से घुमाया जाता है। चित्रानुसार दर्शाया गया है। P(x, y) के निर्देशांक को P(X, Y) , नये निर्देशांक पद्धति XY, में बदला जाता है तब x, y, X, Y नीचे दर्शाया गया है–



$X = x\sec\theta + (y - x\tan\theta)\sin\theta$ and और $Y = (y - x\tan\theta)\cos\theta$

$= x\sec\theta + y\sin\theta - \frac{x\sin^2\theta}{\cos\theta}$ $= y\cos\theta - x\sin\theta$

$\boxed{X = x\cos\theta + y\sin\theta}$ $\boxed{Y = -x\sin\theta + y\cos\theta}$

20. If the axes are rotated through 60^o in anticlockwise direction about origin. Find co-ordinats of point (2, 6) in new co-ordinate axes.

यदि अक्षों को मूल बिन्दु के सापेक्ष वामावर्त दिशा में 60^o कोण से घूर्णन कराया जाता है। नयी निर्देशांक अक्षों की पद्धति में बिन्दु (2, 6) के निर्देशांक ज्ञात कीजिए–

Ans. $(1 + 3\sqrt{3}, -\sqrt{3} + 3)$

Sol. $X = x\cos\theta + y\sin\theta$ $\Rightarrow$ $X = 2\cos 60^o + \cos 60^o = 1 + 3\sqrt{3}$

$Y = -x\sin\theta + y\cos\theta$ $\Rightarrow$ $Y = -2\sin 60^o + 6\cos 60^o = -\sqrt{3} + 3$

21. If axes are rotated through an acute angle in clockwise direction about origin so that equation $x^2 + 2xy + y^2 - 2x + 2y = 0$ becomes free from xy in its new position, then find equation in new position
यदि अक्षों को मूल बिन्दु के सापेक्ष दक्षिणावर्त दिशा में न्यूनकोण से घुमाया जाता है जबकि समीकरण $x^2 + 2xy + y^2 - 2x + 2y = 0$ नयी स्थिति में xy से स्वतंत्र है तब नयी स्थिति में समीकरण है–

Ans. $x^2 + \sqrt{2}y = 0$

Sol. $(x\cos\theta + y\sin\theta)^2 + 2(x\cos\theta + y\sin\theta)(-x\sin\theta + y\cos\theta) + (-x\sin\theta + y\cos\theta)^2$
$- 2x\cos\theta - 2y\sin\theta - 2x\sin\theta + 2y\cos\theta = 0$
$x^2 + y^2 - x^2\sin2\theta + y^2\sin2\theta + 2xy(\cos^2\theta - \sin^2\theta) - 2x(\sin\theta + \cos\theta) + 2y(\cos\theta - \sin\theta) = 0$

$\cos^2\theta - \sin^2\theta = 0 \quad \Rightarrow \quad \tan^2\theta = 1 \quad \Rightarrow \quad \theta = -\frac{\pi}{4} \quad \Rightarrow \quad 2\theta = -\frac{\pi}{2}$

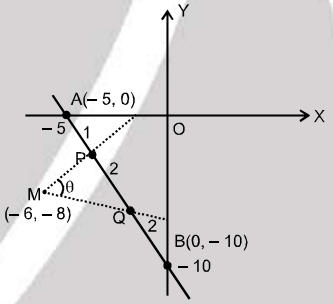
$2x^2 + 2\sqrt{2}\,y = 0 \quad \Rightarrow \quad x^2 + \sqrt{2}\,y = 0$

22. Find the acute angle between two straight lines passing through the point M(−6, −8) and the points in which the line segment 2x + y + 10 = 0 enclosed between the co−ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x−axis to the point of intersection with the y−axis.
बिन्दु M(−6, −8) से गुजरने वाली दो सरल रेखाओं, जो रेखा 2x + y + 10 = 0 के अक्षों के मध्य के अंतःखण्ड को ऐसे बिन्दुओं पर प्रतिच्छेद करती है जो इस रेखाखण्ड को इसके x - अक्ष से प्रतिच्छेद से इसके y - अक्ष से प्रतिच्छेद की दिशा में 1 : 2 : 2 के अनुपात में विभाजित करते हैं, के मध्य का न्यून कोण का मान है।

Ans. $\pi/4$

Sol. $\therefore P \equiv (-4, -2)$ and $Q \equiv (-2, -6)$
$\therefore$ Let slopes of PM and QM be m_1 and m_2 respectively.

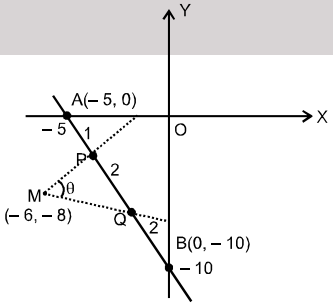
$\therefore \quad m_1 = 3$ and $m_2 = \frac{1}{2}$.



Let 'θ' be the acute angle between PM and QM

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$.

Hindi. $\therefore \quad P \equiv (-4, -2)$ और $Q \equiv (-2, -6)$
$\therefore$ माना PM और QM की प्रवणताएँ क्रमशः m_1 व m_2 हैं।

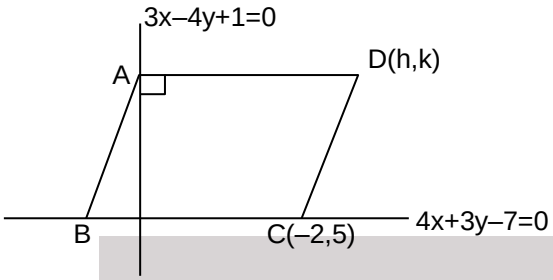


$\therefore \quad m_1 = 3$ तथा $m_2 = \frac{1}{2}$.

माना 'θ' PM तथा QM के बीच का न्यूनकोण है।

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$.

23_. Let A lies on 3x – 4y + 1 = 0. B lies on 4x + 3y – 7= 0 and C is (–2, 5). If ABCD is rhombus, then find locus of D.
माना A, रेखा 3x – 4y + 1 = 0 पर स्थित है तथा, B रेखा 4x + 3y – 7 = 0 पर स्थित है तथा C (–2, 5) है यदि ABCD एक समचतुर्भुज है तब D का बिन्दुपथ है–



DA = DC

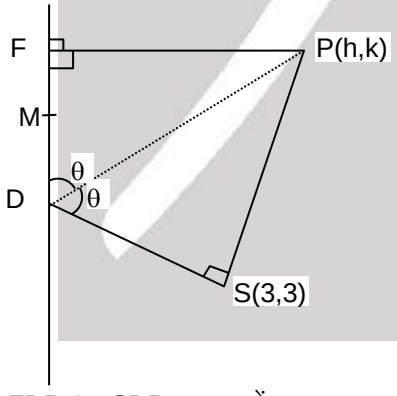
$(h + 2)^2 + (A - 5)^2 = \left(\frac{3h - 4k + 1}{5}\right)^2$

$25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$

Ans. $25((x + 2)^2 + (y - 5)^2) = (3x - 4y + 1)^2$

24_. Let D is point on line ℓ_1 : x + y – 2 = 0 and S(3, 3) is fixed point. ℓ_2 is the line perpendicular to DS and passing through S. If M is another point on line ℓ_1 (other than D), then find locus of point of intersection of ℓ_2 and angle bisector of $\angle$MDS.
माना D, रेखा ℓ_1 : x + y – 2 = 0 पर कोई बिन्दु है तथा S(3, 3) स्थिर बिन्दु है। ℓ_2, DS के लम्बवत् रेखा है तथा S से गुजरती है यदि M, रेखा ℓ_1 पर स्थित कोई अन्य बिन्दु है (D के अलावा) तब $\angle$MDS के कोण अर्द्धक तथा ℓ_2 के प्रतिच्छेद बिन्दु का बिन्दुपथ ज्ञात कीजिए।

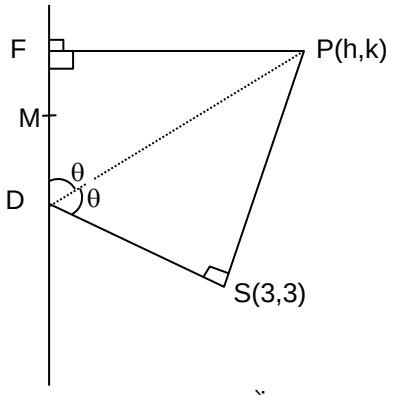
Sol.



In ΔFDP & ΔSDP में
$\angle F = \angle S = 90°$, $\angle FDP = \angle SDP = \theta$ & PD is common
$\Rightarrow \Delta FDP$ & ΔSDP are congurent $\Rightarrow$ PF = PS

$\Rightarrow (h - 3)^2 + (k - 3)^2 = \left(\frac{h + k - 2}{\sqrt{2}}\right)^2$

Hindi.



ΔFDP तथा ΔSDP में

$\angle F = \angle S = 90°$, $\angle FDP = \angle SDP = \theta$ तथा PD उभयनिष्ठ है।

$\Rightarrow \Delta FDP$ तथा ΔSDP सर्वांगसम है $\Rightarrow PF = PS$

$\Rightarrow (h-3)^2 + (k-3)^2 = \left(\frac{h+k-2}{\sqrt{2}}\right)^2$

Ans. $(x-3)^2 + (y-3)^2 = \left(\frac{x+y-2}{\sqrt{2}}\right)^2$

25_. A variable line cuts the line $2y = x - 2$ and $2y = -x + 2$ in points A and B respectively. If A lies in first quadrant, B lies in 4th quadrant and area of ΔAOB is 4, then find locus of

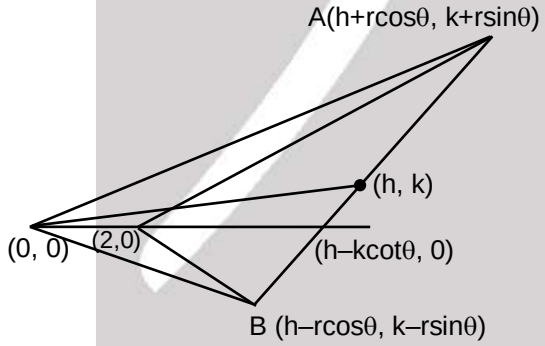
(i) mid point of AB (ii) centroid of ΔOAB

एक चर रेखा, रेखाओं $2y = x - 2$ और $2y = -x + 2$ को A तथा B बिन्दुओं पर काटती है यदि A प्रथम चतुर्थांश मे स्थित है तथा B द्वितीय चतुर्थांश मे स्थित है तथा ΔAOB का क्षेत्रफल 4 है तब बिन्दुपथ ज्ञात कीजिए।

(i) AB का मध्य बिन्दु (ii) ΔOAB का केन्द्रक

Ans. (i) $(x-1)^2 - 4y^2 = 9$ (ii) $(x - 2/3)^2 + 4y^2 = 4$

Sol.



$2(k + r\sin\theta) = (h + r\cos\theta) - 2$

$2(k - r\sin\theta) = -(h - r\cos\theta) + 2$

$\Rightarrow 4k = 2r\cos\theta$ & $h - 2 = 2r\sin\theta \Rightarrow \tan\theta = \frac{h-2}{4k}$

Area $= \frac{1}{2}(h - k\cot\theta)(2r\sin\theta) = 4$

$\Rightarrow (h - k\cot\theta)(h-2) = 8$ $\Rightarrow \left(h - \frac{4k^2}{h-2}\right)(h-2) = 8$

$\Rightarrow (h-1)^2 - (2k)^2 = 9$ $\Rightarrow (x-1)^2 - (2y)^2 = 9$

Locus of centroid is $\left(\frac{3x}{2} - 1\right)^2 - \left(2\left(\frac{3y}{2}\right)\right)^2 = 9$ $\Rightarrow \left(x - \frac{2}{3}\right)^2 - 4y^2 = 4$

26. An equilateral triangle PQR is formed where P (1, 3) is fixed point and Q is moving point on line x = 3. Find the locus of R.
एक समबाहु त्रिभुज PQR बनाया जाता है जहाँ P (1, 3) एक स्थिर बिन्दु है। तथा रेखा Q रेखा x = 3 पर चर बिन्दु है तब R का बिन्दु पथ ज्ञात कीजिए।

Ans. $(x-2) = \pm\sqrt{3}\,(y-3\pm\sqrt{3})$

Sol. Slope of PR की प्रवणता is $(\theta \pm 60^0)$. है।

Let coordinates of माना R के निर्देशांक be (h, k). है।

$$\frac{h-1}{\cos(\theta\pm60^0)} = \frac{k-3}{\sin(\theta\pm60^0)} = 2\sec\theta$$

$h = 1 + 2\cos(\theta \pm 60^0)\sec\theta$

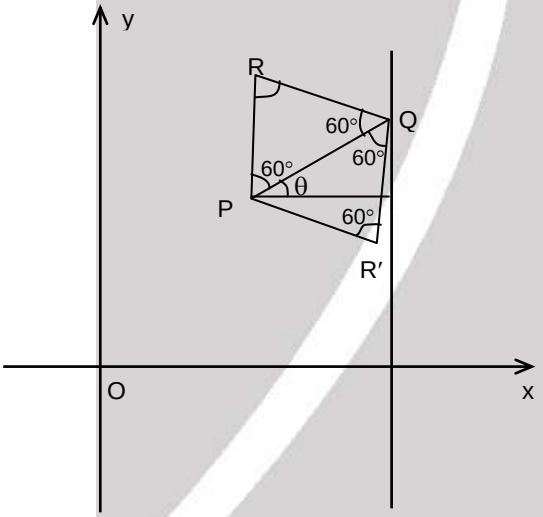
$$\Rightarrow h = 1 + 2\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}\tan\theta\right) \qquad \ldots(1)$$

$\Rightarrow k = 3 + 2\sin(\theta \pm 60^0)\sec\theta$

$$\Rightarrow k = 3 + 2\left(\frac{\tan\theta}{2} \mp \frac{\sqrt{3}}{2}\right) \qquad \ldots(2)$$

Eliminating θ from (1) and (2), (1) और (2) और से θ हटाने पर

$$\frac{h-2}{\sqrt{3}} = \pm\tan\theta$$



$k - 3 \pm \sqrt{3} = \tan\theta$

$$\Rightarrow \frac{h-2}{\sqrt{3}} = \pm(k-3\pm\sqrt{3})$$

Locus is बिन्दु पथ $(x-2) = \pm\sqrt{3}\,(y-3\pm\sqrt{3})$ है।